Session 473:

Arithmetic Sequences and Series as a Bridge to Quadratics

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- Attended UCLA for Undergraduate and Graduate School
- Teachers at Orthopaedic Medical Magnet High School in the Los Angeles Unified School District.
- National Board Certified Teachers
- Park City Math Institute 2013, TwitterMathCamp2015
- Master Teacher Fellows for *Math for America Los Angeles*
  - Project Title: *Creating and Implementing a Pathway to Calculus via Common-Core Aligned Curriculum*

Why We're Here

- We have a "Pathway to Calculus" project with Math for America.

- Designing a 3-year, functions-based, Common Core aligned curriculum with the AP Calculus Exam as one of the end goals.
  - Year 1 Pre-Calculus (Algebra 2)
  - Year 2 Pre-Calculus (Trigonometry and Math Analysis)
  - Year 3 AP Calculus AB/BC

- We are here to share our Unit 3: *Arithmetic Sequences and Series as a Bridge to Quadratic Functions.*

- **Note**: We anticipate these tasks usually take about an hour for our students. In the interest of time, please answer only the questions that interest you (skip the easy ones). We will also recommend certain questions for you to do.

Implementing Tasks

Lesson Approach:

1) Launch

Introduce Scenario through story, picture, story, or combination of these things

2) Investigate

Students work on task in groups while teacher circulate

3) Debrief

Whole class discussion of certain/questions of the task

Individual students share their answers

Teacher use a students' paper and elicit responses from classmates

Task 3.1: Push Up Duck

- <u>Launch</u>: Video (ESPN), do some push ups, talk football
  http://www.youtube.com/watch?v=5ve92hOixGo
- <u>Investigate</u>: Activity 3.1
- <u>Debrief</u>: Discussion
  - **Introduce students to the concept of accumulation.**
  - Spiral concepts including discrete linear functions, representing functions and function notation
  - Lead up to **Arithmetic Sequences and Series**

- <u>Standards Addressed</u>
  - CCSS F-IF.3 Recognize sequences are functions, sometimes defined recursively, whose domain is a subset of integers.
  - CCSS F-LE.2 Construct Linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of relationship, or two input-output pairs.

Year 1 Progression (Algebra II)

- Unit 1: Relations and Functions
- Unit 2: Linear and Piece-wise Linear Functions
- **Unit 3: Arithmetic Sequences and Series**
- Unit 4: Quadratic Functions
- Unit 5: Exponential Functions
- Unit 6: Geometric Sequences and Series
- Unit 7: Equations
- Unit 8: Operations on Functions

Arithmetic Sequence and Series

- **California Math Standards for Algebra II :**
  - ***22.0*** *Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.*
  - ***23.0*** *Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.*
- **Common Core Standards**:
  - ***CCSS F-LE.2*** *Construct Linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of relationship, or two input-output pairs.*
  - ***CCSS F-BF.2*** *Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
  - ***CCSS F-IF.3*** *Recognize sequences are functions, sometimes defined recursively, whose domain is a subset of integers.*

Finding an Explicit Formula

Given the sequence:

5, 7, 9, 11, …

In your groups, show three different ways to write the ***explicit formula*** *(general rule)* for this sequence.

$$a_n =$$

Three Methods

1) $a_n = a_1 + d(n - 1)$

d =common difference

2) $y - y_1 = m\,(y - y_1)$

point (x, y) = (term number, term value)

m = common difference (constant rate of change)

3) $y = mx + b$

m = common difference (constant rate of change)

b = "0^{th} term"

Unit 3 Tasks

- **3.1 Push Up Duck**
- 3.2 Sequences
- 3.3 Arithmetic Sequences
- 3.4 Arithmetic Series
- **3.5 Hot Cheetos Craze / Create Your Own**
- **3.6 Finding a Shortcut**
- **3.7 The Odd Addition**
    - 3.7b: The Even Addition
    - 3.7c: Natural Numbers
- 3.8 Arithmetic Sequences and Quadratic Functions
    - 3.8b Becoming a Factoring Machine
    - 3.8c Solving for x by Factoring
- 3.9 Money for Music
- Quiz
- 3.10 The Marathon
- 3.11 Level Up
- 3:12 Increasing the Pass Rate
- 3.13 The Block Tower
    - 3.13b Using the Quadratic Formula with Series
- Unit 3 Create Your Own: A Real Arithmetic Series
- 3.14 Unit 3 Review
    - 3.14b Sequences and Series Review
- **Unit 3 Exam**

Task 3.5: Hot Cheetos Craze

- Launch: Kids Love Hot Cheetos!!!
- Investigate: Activity 3.5
- Debrief: Discussion
  - Apply Arithmetic Sequences and Series in context
  - Spiral ideas of Arithmetic Sequences and Series, Sigma Notation
  - **Create a need for some sort of shortcut**

- Standards Addressed
  - CCSS F-IF.3 Recognize sequences are functions, sometimes defined recursively, whose domain is a subset of integers.
  - CCSS F-LE.2 Construct Linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of relationship, or two input-output pairs.

Task 3.6: Finding a Shortcut

- Launch: 3.5 Hot Cheetos Craze and CYO
  - There is a need for a shortcut to find the sum of an Arithmetic Series

- Investigate: 3.6 Finding a Shortcut
  - **Guided exploration to find a shortcut**

- Debrief: Discuss the Shortcut then circle back and finish the Hot Cheetos problem.

The sum of the first *k* terms of an Arithmetic Sequence is…

$$\sum_{n=1}^{k} a_n = \frac{k}{2}(a_1 + a_k)$$

Task 3.7: The Odd Addition

- Launch: Lets see if we can use the shortcut we found in 3.6

- **Investigate: 3.7 The Odd Addition**
  - **Sum of first n odd whole numbers**

1 + 3 + 5 + 7 + 9 + …

- Debrief: Discussion what happened and test a few more values.

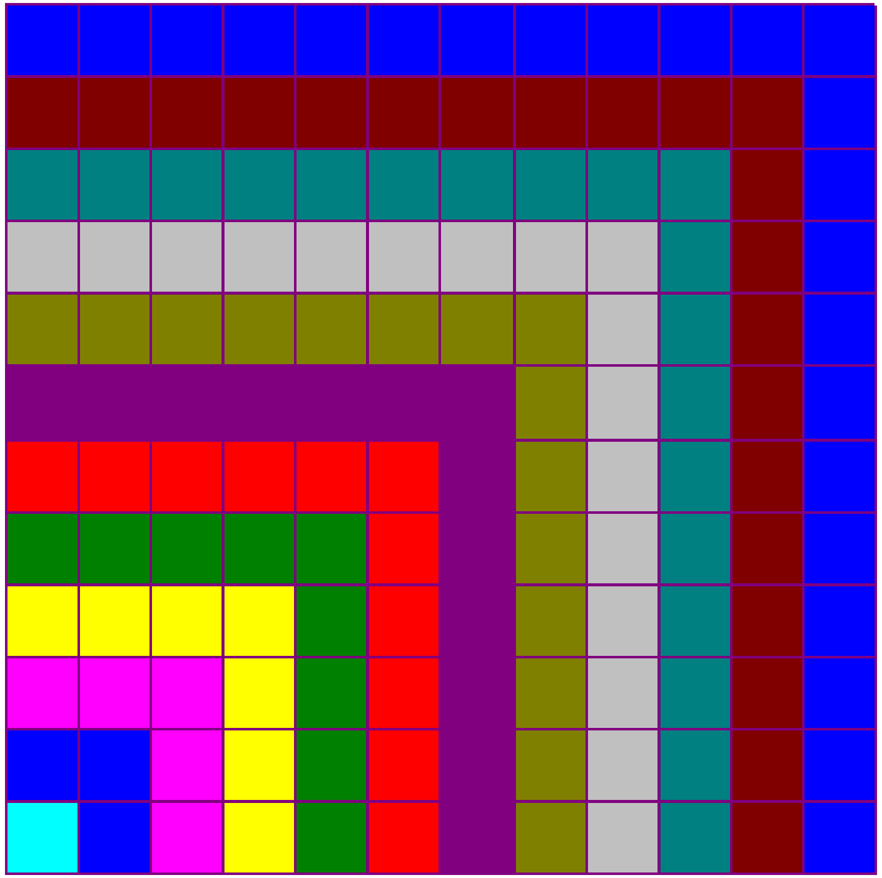
Explicit Formula $a_n = 2n - 1$

First Term $a_1 = 2(1) - 1 = 1$

x^{th} term $a_x = 2x - 1$

$$\sum_{n=1}^{x} 2n - 1 = \frac{x}{2}(1 + 2x - 1) = \frac{x}{2}(2x) = \boldsymbol{x^2}$$

A Visual Representation…



The Connections

- Explicit Formula for Arithmetic Sequence ➔ Linear Function

- Sum of the first *k* terms of an Arithmetic Sequence ➔ Quadratic Function

- We can create and solve linear equations to find the term number of a specific term.

- We can create and solve quadratic equations to find the number of terms required to reach a specific SUM.

Given the Hot Cheeto sequence: $a_n = 20n - 10$

If n is 10 then $a_{10} = 20(10) - 10 = 190$

If $a_n = 190$ then $190 = 20n - 10 \rightarrow n = 10$

If $k = 5$ then $\sum_{n=1}^{k} 20n - 10 = 10k^2 = 10(5)^2 = 250$

If $\sum_{n=1}^{k} 20n - 10 = 250$ then $10k^2 = 250 \rightarrow k = 5$

Unit 3 Exam Part 3: The Card Tower

This was pretty tough for our students … see if you can do it.

Unit 3 Exam Part 3: The Card Tower

4) Create a quadratic function *C* where *C(x)* matches the number of cards in Mr. Goza's structure when it is *x* stories tall.

5) Use factoring to find out how many stories Mr. Goza's structure will have if he uses 210 cards to make it. (Hint: (20)(21) = 420)

6) How many more ***stories taller*** will Mr. Tran's structure be than Mr. Goza's if each of them are given 3 decks of cards to make their structure. Assume each deck has 52 cards.

Questions and Comments?

Contact Information

EVALUATION: Text "*12344 333 a nice comment***"** to **37607**

- Liem Tran
  - liemtran@mfala.org
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*all materials can be found at coast2coast.me https://www.youtube.com/watch?v=RkjdEIe6Ap8&feature=youtu.be

Send your text message to this Phone Number: 37607

poll code for this session

12344 *(1 space)* ___ ___ ___ *(1 space)* _____________

Speaker was engaging and an effective presenter (0-3)

Other comments, suggestions, or feedback (words)

(no spaces)

Speaker was well-prepared and knowledgeable (0-3)

Session matched title and description in program book (0-3)

Example: **12344** 323 Inspiring, good content

Non-Example: **12344** 3 2 3 Inspiring, good content