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Concept 3: Arithmetic Sequences and Series (as a Bridge to Quadratics)

Common Core Standards:

A-SSE 3a
A-CED 1, 2, 3

F-IF 3
F-BF 1a, 2
F-LE 2

Key Vocabulary:

Sequence
Discrete Linear Function
Term
Accumulation
Series
Sigma/Summation Notation
Arithmetic Sequence/Series
Common Difference
Explicit Formula for a Sequence (with a_n notation)
General Rule for a Sequence
n^{th} term
Sum
Evaluate
Quadratic Function
Quadratic Equation
Factoring
Zero Product Property
Quadratic Formula

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3.1: Push Up Duck

Football games at the University of Oregon are always exciting. The Oregon Ducks are known for scoring points, and their mascot is known for keeping track of those points by doing pushups. At the conclusion of each of Oregon's scoring drives the Duck does a pushup for every point the team has scored in the game up to that point. In a high scoring game, this can add up fast!

Watch this clip for info on the Duck: http://www.youtube.com/watch?v=5ve92hOixGo

In 2012, Oregon scored 70 points while beating conference rival Colorado! Here is information from the game: http://scores.espn.go.com/ncf/boxscore?gameId=323012483

GAMEHQ

Scores Pac-12

Final	Final	Final	Final	Final	Final
UCLA 45	COLO 14	10 USC 36	WSU 17	CAL 27	7 ORST 17
ASU 43	2 ORE 70	ARIZ 39	13 STAN 24	UTAH 49	WASH 20

Scoring Summary

FIRST QUARTER				COLO	ORE
ORE	TD	13:13	Kenjon Barner 1 Yd Run (Rob Beard Kick)	0	7
ORE	TD	12:28	De'Anthony Thomas 9 Yd Run (Rob Beard Kick)	0	14
ORE	TD	06:56	Marcus Mariota 5 Yd Run (Rob Beard Kick)	0	21
ORE	TD	01:44	Bralon Addison 16 Yd Pass From Marcus Mariota (Rob Beard Kick)	0	28
SECOND QUARTER				**COLO**	**ORE**
ORE	TD	12:36	Kenjon Barner 24 Yd Run (Rob Beard Kick)	0	35
ORE	TD	11:06	De'Anthony Thomas 73 Yd Punt Return (Rob Beard Kick)	0	42
ORE	TD	06:16	Daryle Hawkins 7 Yd Pass From Marcus Mariota (Rob Beard Kick)	0	49
ORE	TD	00:20	Bryan Bennett 6 Yd Run (Rob Beard Kick)	0	56
THIRD QUARTER				**COLO**	**ORE**
COLO	TD	09:41	Christian Powell 1 Yd Run (Will Oliver Kick)	7	56
COLO	TD	08:02	Christian Powell 20 Yd Run (Will Oliver Kick)	14	56
ORE	TD	03:55	Bryan Bennett 3 Yd Run (Rob Beard Kick)	14	63
ORE	TD	01:00	Bryan Bennett 17 Yd Run (Rob Beard Kick)	14	70

Team Stat Comparison

	COLO	ORE
1st Downs	12	30
3rd down efficiency	2-13	8-12
4th down efficiency	2-3	1-1
Total Yards	246	617
Passing	96	192
Comp-Att	13-27	14-18
Yards per pass	3.6	10.7
Rushing	150	425
Rushing Attempts	33	57
Yards per rush	4.5	7.5
Penalties	5-60	7-85
Turnovers	3	1
Fumbles lost	2	1
Interceptions thrown	1	0
Possession	32:13	27:47

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Answer the following questions regarding Oregon's 70-14 victory against Colorado.

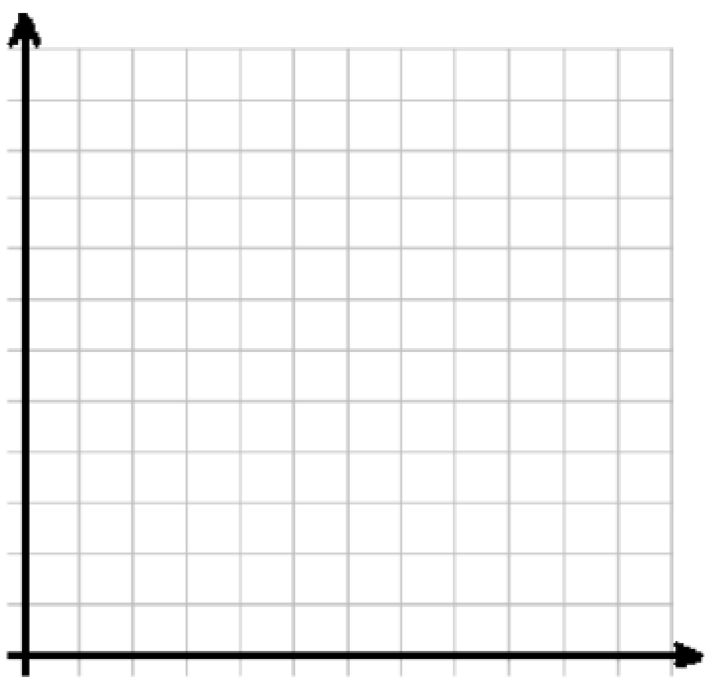
1) How many pushups *did the Duck do* after the first touchdown?

2) How many pushups *did the Duck do* after Oregon scored its third touchdown?

3) How many touchdowns did Oregon score in the Colorado game?

Assume that the function *D(n)* models the number of pushups the Duck did after Oregon scored its n^{th} touchdown during their game against Colorado.

4) Graph the equation ***# of Pushups = D(n)*** on the axis below.



5) True or False: *D is a linear function*. Explain your choice.

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6) Is this function continuous or discrete? Explain why?

7) What is the domain of *D*?

8) What is the range of *D?*

9) What is *D(8)*? What does it represent?

10) How many ***total pushups had the Duck done*** after Oregon scored 3 touchdowns?

11) At Halftime, Oregon had scored 56 points. How many total pushups had the Duck done by then?

12) What was the total number of pushups done by the Duck in the game against Colorado?

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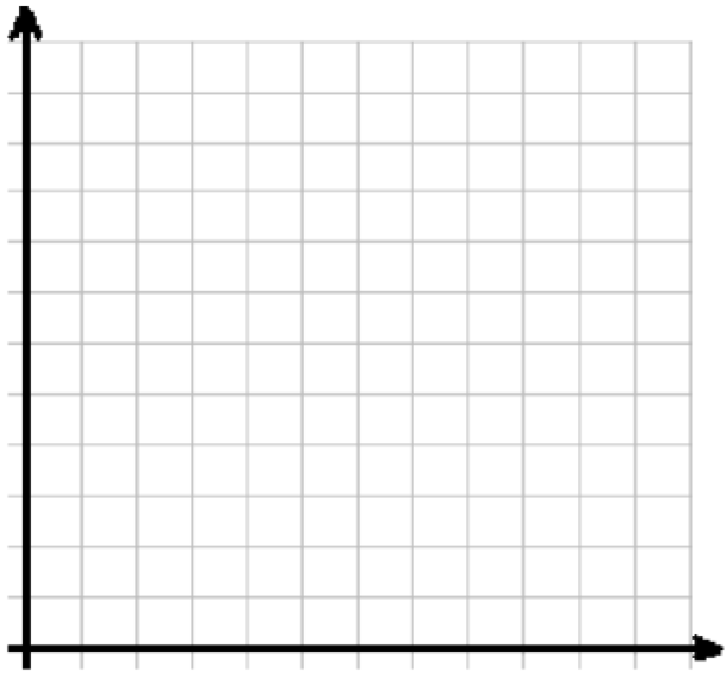
Let's say the function *T(t) models the* ***total number of push-ups*** *the Duck had done after Oregon had scored t touchdowns.*

13) Use the axis below to graph the equation ***# of Pushups = T(t)***. You may want to make a table to keep track of the ordered pairs.



14) True or False: *T is a linear function*. Explain your choice.

15) Using **Sigma Notation**, write an expression to represent the total number of push-ups the Duck had done after Oregon had scored *t* touchdowns.

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3.2: Sequences

A **sequence** is an ordered list of numbers. Each number in the sequence is called a **term**.

For each sequence below, write the next three tems:

1) 2, 4, 6, 8 …

2) 2, -2, 2, -2 …

3) 5, 10, 20, 40 …

4) 1, 4, 9, 16, 25 …

5) 60, 50, 40, 30 …

6) 1, -10, 100, -1000 …

7) 128, 64, 32, 16, …

8) 1, 1, 2, 3, 5, 8, 13 …

9) The sequence in #8 is a famous sequence called the Fibonacci sequence.

   a) What is the 3^{rd} term of the Fibonacci sequence?

   b) What is the 12^{th} term of the Fibonacci sequence?

   c) Explain how to find *the next term* in the Fibonacci sequence.

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Mathematicians use $\boldsymbol{a_n}$ to denote the $\boldsymbol{n^{th}}$ term of a sequence. For example, $\boldsymbol{a_3}$ is the **third term** of a given sequence.

10) For the sequence 3, 5, 7, 9, …

a) What is a_2?

b) What is a_5?

c) a_{10} =______

d) a_{100} =______

11) For the sequence 100, 98, 96, 94, …

a) What is a_3?

b) What is a_6?

c) a_{10} =______

d) a_{50} =______

12) For the sequence 0, -4, -8, -12, …

a) What is a_1?

b) What is a_4?

c) a_{20} =______

d) a_n =____________

13) What do the sequences in #10, 11, & 12 have in common?

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3.3: Arithmetic Sequences

If the terms in a sequence differ by a common amount (called a common difference) the sequence is considered an **Arithmetic Sequence**. Therefore, the terms in an Arithmetic Sequence increase or decrease by a constant amount.

1) Create an increasing Arithmetic Sequence. Explain what makes your sequence Arithmetic.

2) Create a decreasing Arithmetic Sequence. What is the common difference of this sequence?

3) Create a sequence that is not Arithmetic. Explain how you know it's not.

A sequence can be considered a function where the input is the term number in the sequence and the output is the term itself. Then we can use the notation a_n in the same way we use $f(n)$ when dealing with functions. The domain of any sequence is $\{1, 2, 3, \ldots\}$. Can you explain why?

When a sequence follows a specific pattern we can sometimes write an equation to match it. The equation that is used to represent a sequence is called the **explicit formula (or general rule) for the sequence.**

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Since Arithmetic Sequences increase or decrease at a constant rate we can use linear equations to write rules for them.

So for example the equation $\boldsymbol{a_n = 2n + 1}$ represents the sequence $\mathbf{3, 5, 7, 9}, \ldots$

Given each general rule, write the first 5 terms of the sequence:

1) $a_n = 2n + 5$

2) $a_n = -3n$

3) $a_n = 10n + 20$

4) $a_n = -n + 7$

Given the Arithmetic Sequence, use what you know about linear equations to write a general rule to represent the sequence.

5) 10, 13, 16, …

6) 7, 12, 17, …

7) 20, 12, 4, …

8) -6, -8, -10, …

9) 3, 6, 9, …

10) 80, 60, 40, …

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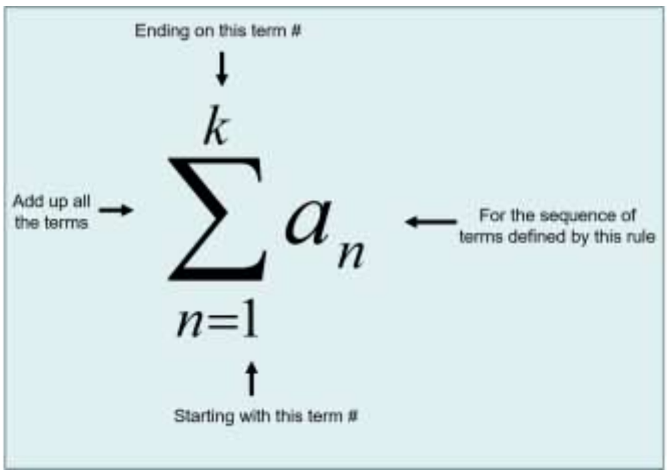
3.4: Series

A **series** is *the sum of the terms* in a sequence. So we can **evaluate** a series by adding terms from a given sequence.

We represent a series with Summation Notation also known as Sigma Notation because we use the Greek letter sigma: $\sum$ (which looks similar to a capital E in English).

The image below describes how we evaluate a series. The value *k* above $\sum$ does not tell us the number of terms we are adding (a common error). It tells us the term number that will be the last in the sum. So if there is a 4 above the $\sum$ we stop adding terms when we get to the 4^{th} term in the sequence.



Evaluate:

1) $\sum_{n=1}^{5} 4n+1$

2) $\sum_{n=1}^{4} n^2+1$

3) $\sum_{n=3}^{9} 2n+4$

4) $\sum_{n=1}^{3} -3n-5$

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5) $\sum_{n=1}^{4} 2^{n}$

6) $\sum_{n=1}^{4} -5n+25$

An **Arithmetic Series** is the sum of the terms in an Arithmetic Sequence.

7) Which of the series in #1 – #6 are Arithmetic Series? How do you know?

Write each of the sums below using Summation/Sigma Notation:

8) $7+9+11+13+15+17$

9) $20+25+30+35+40$

10) $22+19+16+13+11+8+5+2-1$

11) $-20-30-40-50$

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3.5: Hot Cheetos Craze

Kids these days love Hot Cheetos. Last time I saw an Ortho student at Target, this was their cart:

Ms. Weeks, who fears the worst, thinks kids are eating more and more junk food. She predicts that a typical Ortho student eats their first 10 small bags of Hot Cheetos in 1st grade, eats 30 small bags in 2nd grade, eats 50 small bags in 3rd grade and continues this pattern of additional consumption until they graduate high school.

1) How many bags of Hot Cheetos does a given student eat in 5th grade, according to Ms. Weeks?

2) The number of Hot Cheetos a student eats each year can be thought of as a sequence. Write the first 5 terms of the sequence.

3) What type of sequence is this? How do you know?

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4) Find the explicit formula ($a_n =$ ______) that matches this sequence.

5) According to your formula what is a_9? What does it represent in this scenario?

6) How many small bags of Hot Cheetos does Ms. Weeks predict a typical Ortho student will eat in his/her senior year? Does this sound reasonable? Why or why not?

7) According to Ms. Weeks, how many total bags of Hot Cheetos will a student have eaten by the time they graduate from Ortho?

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Mr. Warren says that, without proper health education, this pattern of Hot Cheeto consumption can continue far beyond high school graduation. Say a student continued to increase their Cheeto eating for sixty years (until about the time he/she retired).

8) Does Mr. Warren's fear seem reasonable to you? Why or why not?

9) Create an expression using Sigma Notation to find how many small bags of Hot Cheetos the student will have eaten by the time they retire after a total of 60 years of Hot Cheeto eating.

10) Explain why no one wants to evaluate the expression in #9.

11) Take a few minutes to estimate the sum from #8. Show some mathematical evidence or reasoning that supports your estimate.

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Unit 3 CYO 1: Creating a Series

Create your own Arithmetic Sequence. Use Sigma Notation to create an expression that represents the sum of the first 20 terms. Find the sum on another sheet of paper. Don't show anyone!

Trade with a partner and calculate the sum of their series. Race if you want to!

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3.6: Finding the Shortcut

Lets think about the most basic twenty term Arithmetic Series out there: the on that adds the first 20 “natural numbers”.

1 + 2 + 3 +…

1) What rule matches the sequence 1, 2, 3, …

2) Use sigma notation to create an expression that represents the sum of the first 20 terms of this sequence.

3) Instead of adding the numbers one by one, write out the entire sum so we can look for patterns that will make it easier to add. Be creative. Do you see anything?

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4) Add the first and the last term of the sum. What do you get?

5) Add the second and second to last term. What do you get?

6) How many such pairs are there?

7) What is the sum of the first 20 terms? (Don't add them one by one!)

8) This will help you later:

Explain where the 21 came from.

Explain where the 10 came from.

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9) Quick, let's try another: $\sum_{n=1}^{20} 2n+4$ using the same pattern.

What is the sum of the first and last term?

How many such pairs are there?

What is the sum of all twenty terms?

10) Explain how to find the sum of the first 20 terms of any Arithmetic Series.

11) Let's use a similar process to evaluate $\sum_{n=1}^{50} 2n$.

What is the sum of the first and last term?

How many such pairs are there?

What is the sum of all 50 terms?

12) How is this similar to the 20 term series?

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13) Generalize what you did in #9 and #11 to explain how to find the sum of the first *k* terms of any Arithmetic Series.

14) Can you write a formula for $\sum_{n=1}^{k} a_n$? It's possible and would be cool if you do!

15) Use your formula to evaluate:

a) $\sum_{n=1}^{30} 3n+5$

b) $\sum_{n=1}^{100} -2n+10$

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3.7: The Odd Addition

1) Think about the sequence of odd natural numbers starting with one. Write the first 5 terms of that sequence. What kind of sequence is it?

2) Use the terms in number 1 to complete the table below:

x	The sum of the first x odd natural numbers
1	
2	
3	
4	
5	

3) What pattern do you notice in the table?

4) Use the pattern to predict the sum of the first 10 odd numbers.

5) Say that there is a function T such the $T(x)$ is the sum of the first x odd natural numbers. Then according to the table…

$T(x)$ = _______

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The result in #5 is simple and fantastic. The sum of the terms in the sequence 1, 3, 5, … always adds to the square of the number of terms. $T(x) = x^2$ is the most basic ***Quadratic Function***, a type of function we will explore further. Let's take a minute to prove this equation to be true.

6) Write a rule to match the sequence 1, 3, 5, …

$a_n =$

7) Use summation notation to create an expression that represents the sum of the first 20 terms of the series 1 + 3 + 5 + …

8) Use the equation for the sum of the first *k* terms: $\sum_{n=1}^{k} a_n = \frac{k}{2}(a_1 + a_k)$ to evaluate the expression you created in #7.

9) Use the same process from #7 & #8 to write a simplified expression for the sum of the **first *x* terms** of the series 1 + 3 + 5 + … (Hint: Set *k* = *x* and simplify the resulting expression.)

Your answer to #9 should be the same answer you got in #5. Check that it is. Hooray Math!

10) Using the function for #5, what is *T(100)*? What does the answer tell us?

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3.7b Adding the Evens

1) Count to twenty by twos in your head. Write the sequence you just counted allowing it to continue infinitely.

2) What kind of sequence is it? How do you know?

3) Write a rule for the sequence from #2.

4) Find the sum of the first one hundred even integers (starting with 2).
Hint: Use the rule from #2 and sigma notation.)

5) Write an expression you could use to find the sum of **the first *x* even integers** in summation notation.

6) Evaluate your expression from #5 by substituting *x* into the shortcut for an Arithmetic Series.

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7) **Call *T(x)* the function that represents the sum of the first *x* even integers. Then according to your previous work:**

***T(x)* =**

8) Use the equation to find *T(100)*. What does your answer represent? (You've seen this answer before!)

8) Use your equation for *T(x)* to find out how many even integers you have to add to get a sum of…

a) 20

b) 110

9) Is it possible to get a sum of 100 by adding up even integers starting with 2? Why or why not?

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3.7c: The Sum of the First *n* "Natural Numbers"

1) Count to ten in your head. Write the sequence you just created allowing it to continue infinitely.

2) What kind of sequence is it? How do you know?

3) Find the sum of the first one hundred "natural numbers" (starting with 1).

4) Write an expression you could use to find the sum of the first *x* natural numbers in summation notation.

5) Evaluate your expression from #5 by substituting *x* into the shortcut for an Arithmetic Series.

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6) Call $T(x)$ the function that represents the sum of the first x natural numbers. Then according to your previous work:

T(x) =

7) Use your equation for $T(x)$ to find out how many natural numbers you have to add to get a sum of…

a) 45

b) 300

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3.8: Arithmetic Sequences & Quadratic Equations

For each sequence, write a quadratic equation $T(x)$ to represent the sum of the first x terms in that sequence. Then use that equation to find $T(10)$.

1) 10, 12, 14, …

2) 8, 14, 20, …

3) 100, 80, 60, …

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4) 30, 40, 50, …

5) 24, 28, 32, …

6) -30, -25, -20, …

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3.8b: Become a Factoring Machine

Solve for *x* by factoring. Show your work on a separate sheet of paper.

1. $x^2 - 14x + 45 = 0$
2. $x^2 + 17x + 60 = 0$
3. $x^2 - 18x + 80 = 0$
4. $x^2 - 10x + 16 = 0$
5. $x^2 - 6x + 9 = 0$
6. $x^2 - 7x + 6 = 0$
7. $x^2 + 20x + 99 = 0$
8. $x^2 + 3x - 18 = 0$
9. $x^2 - 3x - 88 = 0$
10. $x^2 - 16x + 48 = 0$
11. $x^2 + 11x + 30 = 0$
12. $x^2 - 14x + 33 = 0$
13. $x^2 + x - 30 = 0$
14. $x^2 - 3x - 70 = 0$
15. $x^2 + 8x - 9 = 0$
16. $x^2 - 16x + 55 = 0$
17. $x^2 + 6x - 72 = 0$
18. $x^2 + 5x - 50 = 0$
19. $x^2 + 10x + 24 = 0$
20. $x^2 + 6x - 16 = 0$
21. $x^2 - 5x + 4 = 0$
22. $x^2 - 16x + 60 = 0$
23. $x^2 + 8x - 20 = 0$
24. $x^2 - 4x + 3 = 0$
25. $x^2 + 2x - 35 = 0$
26. $x^2 - 5x - 24 = 0$
27. $x^2 - 6x + 8 = 0$
28. $x^2 + x - 90 = 0$
29. $x^2 - 100 = 0$
30. $x^2 - 16 = 0$

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3.8c: Solve for *x* by Factoring

1) $x^2 + 8x + 15 = 0$

2) $x^2 + 9x + 14 = 0$

3) $x^2 + 11x + 30 = 0$

4) $x^2 + 7x = -6$

5) $x^2 - 2x - 15 = 0$

6) $x^2 + 5x - 24 = 0$

7) $x^2 + 3x = 40$

8) $x^2 - 5x - 50 = 0$

9) $x^2 - 9x + 20 = 0$

10) $2x^2 - 12x + 16 = 0$

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11) $x^2 - 9x = 22$

12) $5x^2 + 35x = -50$

13) $8x^2 + 10x + 3 = 0$

14) $10x^2 + 13x + 4 = 0$

15) $6x^2 + 7x - 5 = 0$

16) $9x^2 + 9x + 10 = 8$

17) $12x^2 + 15x + 3 = 0$

18) $6x^2 + 10x - 4 = 0$

19) $4x^2 - 16x + 7 = 0$

20) $10x^2 + 11x + 8 = 5$

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3.9: Money for Music

The music club at a school needs $300 to buy new equipment. So far they have $60. To raise money they decide to have a concert. For every ticket sold to their after-school concert the music club makes $2 profit. On the first day they sold 3 tickets and each day after they sold 2 more tickets than the day before.

1) How much profit did the music club make on day 1?

2) How many tickets did the music club sell on day 3?

3) How much profit did the music club make on day 3?

4) Create a sequence to match the amount of profit the music club makes on day 1, 2, 3, etc.

5) What kind of sequence is this? How do you know?

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6) How much total profit will the music club have made after one week?

7) Using the shortcut for the sum of an Arithmetic Series, create a quadratic function we could use to represent the total profit that the music club will have made after d days.

8) Use your quadratic equation to find the number of days it will take for the music club to make the money they need for their new equipment?

9) How many students do they have to get to buy tickets to make enough money to get the new equipment?

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Quiz: Arithmetic Sequences and Series (Version A)

1) What is the difference between a sequence and a series?

2) Given the sequence of numbers 8, 10, 12…

   a) Write a rule to match this sequence.

   b) Find the 30^{th} term of the sequence.

   c) Find the sum of the first 30 terms.

   d) Create a function *T(x)* that represents the sum of the first *x* terms in this sequence.

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e) Find *T(40)*. What does your answer represent?

f) Find x when $T(x) = 120$. What does your answer represent?

3) How do you know if a series or sequence is Arithmetic?

4) Solve for x: $x^2 - 9x = 22$

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Quiz: Arithmetic Sequences and Series (Version B)

1) What is the difference between a sequence and a series?

2) Given the sequence of numbers 10, 12, 14…

   a) Write a rule to match this sequence.

   b) Find the 50th term.

   c) Find the sum of the first 50 terms.

   d) Create a function *T(x)* that represents the sum of the first *x* terms in this sequence.

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e) Find $T(30)$. What does your answer represent?

f) Find x when $T(x) = 220$. What does your answer represent?

3) Solve for x: $x^2 - 9x = 20$

4) How do you know if a series or sequence is Arithmetic?

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3.10: The Marathon

Mr. Lough loves running, music and New Orleans so this winter he signed up for the New Orleans Rock ‘n’ Roll Marathon. Sadly, however, he’s a little out of shape. He needs to start training for the big run (Marathons are 26 miles). This week will be Week 1 in Mr. Lough’s marathon training. The table below shows how much he will be running each day this week.

Day	Monday	Tuesday (Low Speed)	Wednesday	Thursday (Hills)	Friday	Saturday (Distance)	Sunday (High Speed)
Miles	Rest	4	Rest	3	Rest	6	3

Each week following Week 1, Mr. Lough plans to run an additional half mile on weekday runs and an additional mile on every Saturday Run (when he works on endurance). On Sunday’s when he works on speed training he will always run 3 miles.

On the last week of his training Mr. Lough will totally change his plan and do two light runs. He will do a 6 mile run on Tuesday and a 4 mile run on Thursday before the Marathon on Sunday (he won’t run on Saturday, he’ll just eat a lot of carbs).

In order to help motivate Mr. Lough, Mr. Tran and I told him we would take him to a steak dinner after he reaches a total of 100 miles ran and we got Mr. Mata to promise him a new pair of running shoes when he gets to 250 miles.

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Answer the following questions based on the information from the first page:

1) How many miles is Mr. Lough running today?

2) How many *total miles* will Mr. Lough have run after Week 1?

3) How many miles will he run 2 weeks from this Saturday?

4) Thanksgiving is Thursday November 27th. How many miles will he run on that day? What type of training will it be?

5) How many miles will Mr. Lough run during Week 2? How many ***total miles*** will he have run after week 2?

6) Write an equation for *t(w)* the number of miles Mr. Lough runs on Tuesdays on week *w*. Keep in mind, $t(1) = 4$.

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7) Use *t(w)* to find the number of miles Mr. Lough will run on Tuesday during the 10^{th} week of his training.

8) Create equations for *r(w)*, *s(w)* and *n(w)* the number of miles Mr. Lough runs during week *w* on Thursday (*r*), Saturday (*s*), and Sunday (*n*).

9) Use your equation to find *s(13)*. Why is this answer especially significant?

10) Mr. Lough will run exactly 8 miles three times during his training. On what dates will the 8-mile runs occur?

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11) Write an equation for *m(w)*, the number of miles Mr. Lough runs on week *w*.

12) Use *m(w)* to determine how many miles Mr. Lough will run on Week 7.

13) There will be one week when Mr. Lough runs exactly 30 miles. *Set up and solve an equation* to find which week it is.

14) Notice that *t(w), r(w), s(w)* and *n(w)* are all in terms of the same variable. Because of this, we can easily add them together.

Find *t(w)+r(w)+ s(w)+n(w)*

15) What do you notice about the sum in #14 (after you simplify)?

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16) Because we start at Week 1, the function *m(w)* represents a "sequence." List the first 8 terms of the sequence defined by *m(w)*.

17) What is the last term of the sequence defined by *m(w)*?

18) What kind of sequence is it? How do you know?

19) What is the sum of the first 3 terms of the series defined by *m(w)*?

20) Write the sum of the first 5 terms <u>in summation notation</u>. Evaluate the sum.

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21) When Mr. Lough finishes the marathon (which is 26 miles), how many miles will he have run in total since he started training this Tuesday?

22) Let $T(x) = \sum_{w=1}^{x} m(w)$. Fill out the following table (you have done most of these already):

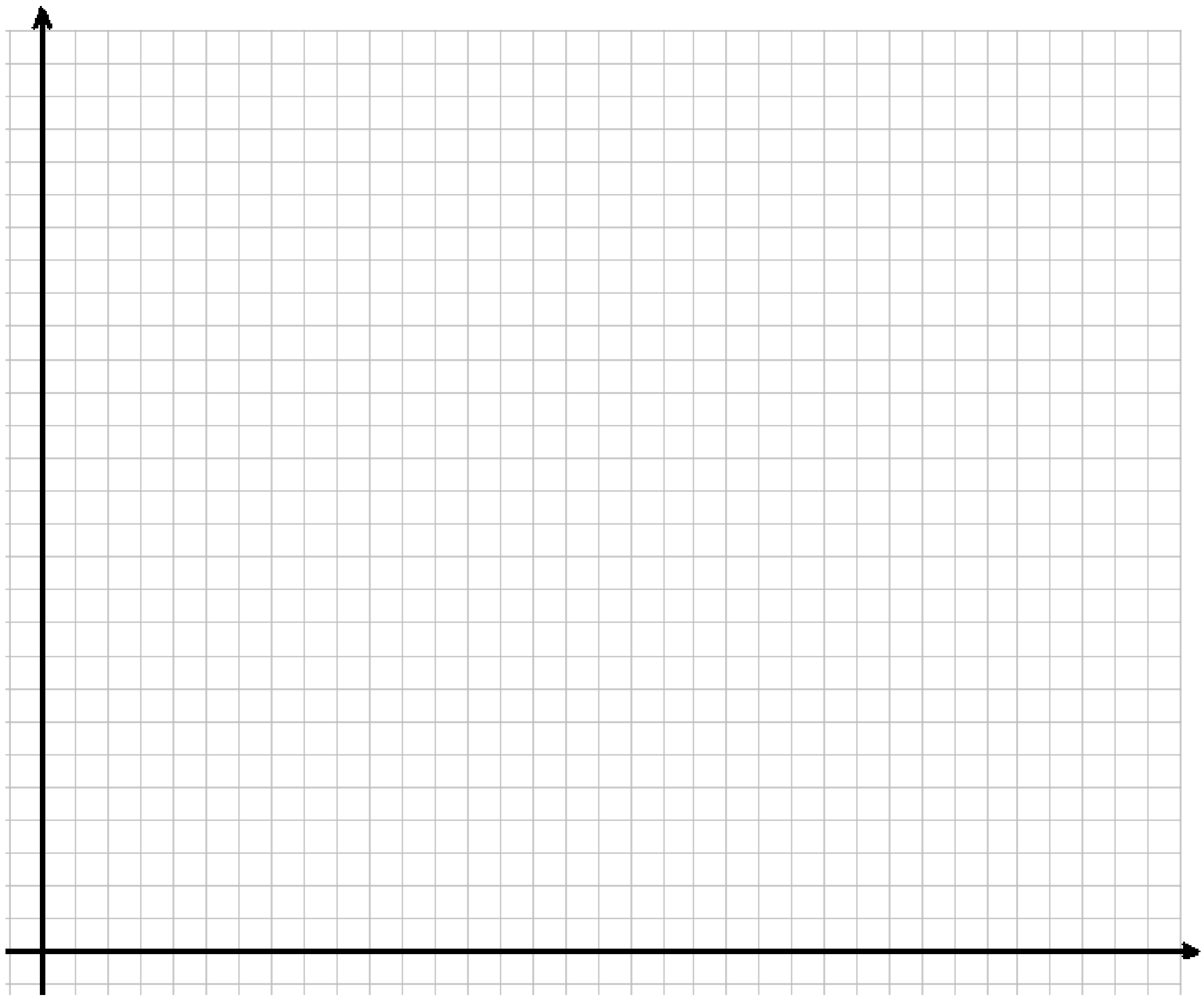
x	*T(x)*
1	
2	
3	
4	
5	

23) What does *T(x)* represent?

24) Assume that *T(0)* = 0. Why does this make sense?

Name: Date: Period:

25) Graph *T(x)*. Think about whether it is continuous or discrete.



26) Is *T(x)* continuous or discrete? Explain your answer.

27) What is the domain of *T(x)*?

Name: Date: Period:

28) **We can write an equation to model *T(x)* by using the shortcut for the sum of an Arithmetic Series: if we allow the "last term" to be the variable *x*. In other words, use** $T(x)=\sum_{w=1}^{x} m(w)=\frac{x(a_1+a_x)}{2}$. **Show your work to create this equation.**

29) Set your equation for *T(x)* equal to 100 then solve for *x*. You should get 2 answers, one of which is *not* in the domain. How does your answer that *is in the domain* affect Mr. Tran?

30) Use your equation for *T(x)* to find the week when Mr. Mata will have to buy Mr. Lough that new pair of shoes. Show your work!

Extension: Say Mr. Lough decides not to run the marathon and just keep with his training program until he gets to 1000 miles!

Name: Date: Period:

1) Use *T(x)* to find the number of weeks it will take for Mr. Lough to run 1000 miles.

2) How many miles will he run on the last week to get to 1000?

3) How many miles will his Saturday run be that week?

4) On the way to 1000 miles how many of his miles will be run on Saturdays?

5) What percentage of his miles will be run on Saturdays?

6) How does the answer in #5 compare to percentage of miles he ran on Saturday during the first week? What causes the difference?

Name: Date: Period:

3.11: Level Up

A new game on the App Store called ***Clash of Clones*** is getting really popular. In each level you have to go around defeating armies of clones of annoying pop-culture celebrities. Level one, for example, requires you to defeat 25 Drake clones before you pass the level. In level two, there are 35 Miley Cyrus clones to kill. In each of the following levels there are ten more enemies than there were in the level before.

1) Create a sequence to match the number of enemies in each level of ***Clash of Clones***. Then write a rule to match the sequence. How many clones would you have to kill in the 100^{th} level? Who would the clones be?

2) Create a quadratic function *C(x)* to model the total number of clones you have killed after completing *x* levels in ***Clash of Clones***. How many clones will you have killed after 8 levels? After how many levels will you have defeated 300 clones?

3) You reach "Clone Conqueror Status" after you kill the 700^{th} clone. After which level does this occur? How did you kill that 700^{th} clone?

Name: Date: Period:

3.12: Increasing the Pass Rate

This year 22 students passed the AP Calculus Test. Mr. Goza predicts that every year going forward, 8 more Ortho Students will pass the AP Test.

1) What sequence matches this scenario? Write the first 3 terms and **a rule** for the sequence.

2) How many students will pass the AP Calculus Test in year 4?

3) How many ***total students*** (in Ortho History) will have passed the AP Calculus Test after year 5 (assuming that last year counts as year 1).?

4) Write a quadratic function to model the total number of Ortho students *T(x)* who will have passed the AP Calc Test after year *x* (assuming that last year counts as year 1).

Name: Date: Period:

5) Use your function to find out the year that the 400th Ortho student will pass the AP exam.

6) Use the Quadratic Formula determine the year that the 100th student will pass the test.

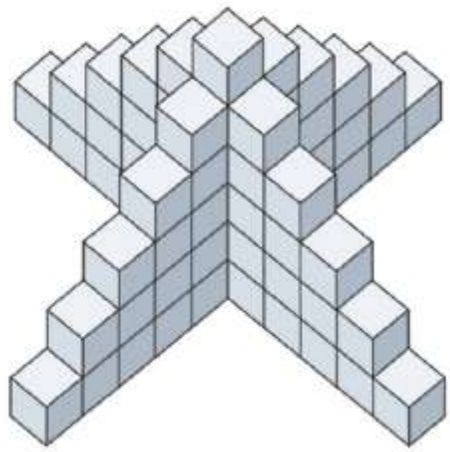
7) How does your answer match the scenario?

Name: Date: Period:

3.13: The Block Tower

(A modified version of "Skeleton Tower" from Illustrative Mathematics)

There are six levels in the tower below.



1) How many cubes are there in this tower?

2) Explain the process you used to count the blocks.

3) How many cubes would it take to build a tower like this that was 12 levels high?

4) How would you calculate the number of cubes in a tower that was *x* levels high?

Name: Date: Period:

5) Assume the quadratic function *c(x)* models the total number of cubes used to build a tower that is *x* levels high. Write an equation for *c(x)*.

6) Use your equation to find the number of levels that you could build with 28 cubes.

7) Use your equation to find the number of levels that you could build with 190 cubes.

8) Is it possible to build a tower like this with 500 blocks? Explain why or why not.

9) How many levels could you build with 1000 blocks? (Use the Quadratic Formula to confirm your answer.)

Name: Date: Period:

3.13b: Using the Quadratic Formula with Series

1) Given $2 + 10 + 18 + \ldots$

a) Find the sum of the first 3 terms.

b) Find the sum of the first 200 terms.

c) Find the sum of the first *x* terms.

d) When will the sum be 90? Try to solve using ***factoring***.

e) When will the sum exceed 10,000? Try to find this using the ***Quadratic Formula***.

Name: Date: Period:

1) Given $9 + 11 + 13 + \ldots$

a) Find the sum of the first 5 terms.

b) Find the sum of the first 1000 terms.

c) Find the sum of the first x terms.

d) When will the sum be 105? Try to solve using ***factoring***.

e) When will the sum exceed 3,000? Try to find this using the ***Quadratic Formula***.

Name: Date: Period:

3) Given $30 + 20 + 10 + \ldots$

a) Find the sum of the first 4 terms.

b) Find the sum of the first 500 terms.

c) Find the sum of the first *x* terms.

d) When will the sum be 0? Try to solve using ***factoring***.

e) Check your work to part "d" by writing out the sum to check that it's zero.

f) When will the sum get below -5,000? Try to find this using the ***Quadratic Formula***.

Name: Date: Period:

Unit 3 CYO: A Real Arithmetic Series

Create your own drawing/picture/figure (like the block tower) that could match up to an Arithmetic Series. Make sure that the sum of the terms actually means something!

Work together to create a Quadratic Function to match the sum of the first x terms of the sequence in each of your problems.

In words, explain what your quadratic function models.

Name: Date: Period:

3.14: Unit 3 Review

Given the rule, write the first 3 terms of the sequence.

1) $a_n = 2n + 1$

2) $a_n = -5n - 2$

Given each sequence write the next term and *then give the rule a_n that defines it.*

3) 7, 11, 15, ____, …

4) 2, 6, 10, ____, …

5) 36, 24, 12, ____, …

6) 20, 15, 10____, …

Evaluate each:

7) $\sum_{n=2}^{4} n^2 + 4n + 1$

8) $\sum_{n=1}^{x} 2n + 12$

9) $\sum_{n=1}^{40} 4n - 3$

10) $\sum_{n=1}^{t} 12n + 10$

Name: Date: Period:

11) Find the 100th term in the sequence 2, -3, -8, …

12) Evaluate:

a) $\sum_{n=3}^{5} n^2 - 5$

b) $\sum_{n=1}^{x} 4n + 8$

13) If the first term of an arithmetic sequence is 4 and the third term is 16, what is the ninth term?

14) Write each in summation/sigma notation. Then find the sum of the first 20 terms.

a) 9 + 13 + 17 + 21 + …

b) 35 + 30 + 25 +10 + …

15) What is the difference between a sequence and a series?

Name: Date: Period:

16) Imagine the list of numbers starting with 20 that increases by 6 every term.

a) Give the sequence that matches this list.

b) True or False. This sequence is an arithmetic sequence.

Justify your answer.

c) Assume the quadratic function *T(x)* gives the sum of the first *x* terms of this sequence. Write an equation for *T(x).*

d) ***Use your equation*** to find the sum of the first 40 terms.

17) Solve for x:

a) $27 = x^2 - 6x$

b) $8x^2 - 3x - 5 = 0$

Name: Date: Period:

Unit 3 Exam: Arithmetic Sequences and Series as a Bridge to Quadratics
(Version A)

1) Find the 300th term in the sequence 10, 4, -2, … (2 points)

2) Evaluate: (2 each)

a) $\sum_{n=1}^{4} n^2 + 1$ b) $\sum_{n=1}^{t} 10n + 6$

3) Assuming the series 10 + 18 + 26 + … continues for 200 terms. Find the sum. (3)

4) Solve for *x* by factoring: (2 each)

a) $x^2 + 10x + 21 = 0$ b) $6x^2 + 7x = 10$

5) What is the difference between a sequence and a series? (1)

Name: Date: Period:

6) Imagine the list of numbers starting with 8 that increases by 10 every term.

a) Give the sequence that matches this list. (1)

b) What type of sequence is this? Justify your answer. (2)

c) Assume the quadratic function *T(x)* gives the sum of the first *x* terms of this sequence. Write an equation for *T(x).* (3)

d) ***Use your equation*** to find the sum of the first 40 terms. (2)

e) ***Use your equation*** to find how many terms you would need to add to get a total of 140. (3)

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Name: Date: Period:

7) Kids these days love Hot Cheetos. Last time I saw an Ortho student at Target, this was their cart:

A typical Ortho Student eats their first 10 small bags of Hot Cheetos in 1^{st} grade and then continues to eat twenty more bags every year until they graduate high school.

a) What sequence matches this scenario? (1)

b) Create an expression using Sigma Notation to find how many small bags of Hot Cheetos a student will have eaten by graduation (12^{th} grade). (2)

c) Write a quadratic equation *C(g)* to match ***total number*** of Hot Cheetos a student will have eaten after finishing grade *g*. (2)

Name: Date: Period:

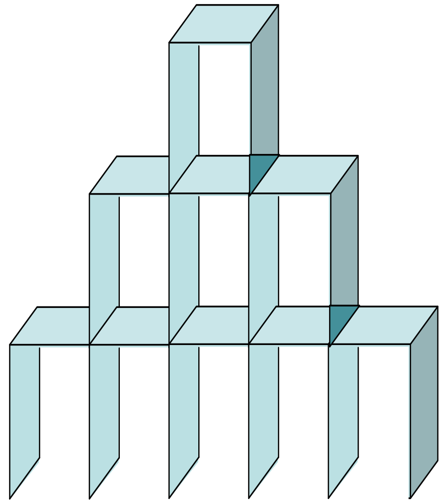
d) Use your equation to find out what grade a student will have completed when he/she has eaten a total of 640 bags of Hot Cheetos. (2)

e) Each small bag of Hot Cheetos has 170 calories (yup, its like drinking fatty oil mixed with sodium). If that is the case, how many calories will a typical Ortho student consume over the course of their education (Grades 1 – 12)? (2)

EC) Does a typical Ortho Student eat more bags of Hot Cheetos during his/her time at Ortho, or in the 8 years of school leading up to High School? Justify your answer.

8) Mr. Tran and Mr. Goza have each begun constructing card houses as shown below. Mr. Tran, who loves triangles, has created the structure on the left, and Mr. Goza has opted for rectangular shapes as shown on the right.

Name: Date: Period:

a) Who uses more cards to create a 4 story version of their figure? Justify your answer with mathematical evidence. <u>(3)</u>

b) How many cards would it take Mr. Tran to make his card structure 10 stories tall? How about 100 stores tall? <u>(3)</u>

c) If Mr. Goza builds his structure to a height of 20 stories, how many cards will he need for the bottom level? <u>(2)</u>

d) Create a quadratic function *C(x)* to match the number of cards in Mr. Goza's structure when it is *x* stories tall. <u>(2)</u>

Name: Date: Period:

e) Use factoring to find out how many stories Mr. Goza's structure will have if he uses 210 cards to make it. (Hint: (20)(21) = 420) (2)

f) How many more ***stories taller*** will Mr. Tran's structure be than Mr. Goza's if each of them are given 3 decks of cards to make their structure. Assume each deck has 52 cards. (Your answer to # 2 should help! You may want to use the Quadratic Formula for Mr. Goza's structure.) (4)

Unit 3 Exam: Arithmetic Sequences and Series as a Bridge to Quadratics (Version B)

1) Find the 400^{th} term in the sequence 12, 7, 2, … (2 points)

Name: Date: Period:

2) Evaluate: (2 each)

a) $\sum_{n=1}^{4} n^2 - 1$ b) $\sum_{n=1}^{t} 10n + 14$

3) Assuming the series 6 + 14 + 22 + … continues for 200 terms. Find the sum. (3)

4) Solve for *x* by factoring: (2 each)

a) $x^2 + 12x + 32 = 0$ b) $6x^2 - 7x = 10$

5) What is the difference between a sequence and a series? (1)

6) Imagine the list of numbers starting with 12 that increases by 10 every term.

a) Give the sequence that matches this list. (1)

Name: Date: Period:

b) What type of sequence is this? Justify your answer. (2)

c) Assume the quadratic function *T(x)* gives the sum of the first *x* terms of this sequence. Write an equation for *T(x)*. (3)

d) ***Use your equation*** to find the sum of the first 50 terms. (2)

e) ***Use your equation*** to find how many terms you would need to add to get a total of 160. (3)

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7) Kids these days love them some Hot Cheetos. Last time I saw an Ortho student at target, this was their cart:

Name: Date: Period:

A typical Ortho Student eats their first 10 small bags of Hot Cheetos in 1st grade and then continues to eat twenty more bags every year until they graduate high school.

a) What sequence matches this scenario? (1)

b) Create an expression using Sigma Notation to find how many small bags of Hot Cheetos will a student have eaten by graduation (12th grade). (2)

c) Write a quadratic equation *C(g)* to match ***total number*** of Hot Cheetos a student will have eaten after finishing grade *g*. (2)

Name: Date: Period:

d) Use your equation to find out what grade a student will have completed when he/she has eaten a total of 490 bags of Hot Cheetos. (2)

e) Each small bag of Hot Cheetos has 170 calories (yup, its like drinking fatty oil mixed with sodium). If that is the case, how many calories will a typical Ortho student consume over the course of their education (Grades 1 – 12)? (2)

EC) Does a typical Ortho Student eat more bags of Hot Cheetos during his/her time at Ortho, or in the 8 years of school leading up to High School? Justify your answer.

8) Mr. Tran and Mr. Goza have each begun constructing card houses as shown below. Mr. Tran, who loves triangles, has created the structure on the left, and Mr. Goza has opted for rectangular shapes as shown on the right.

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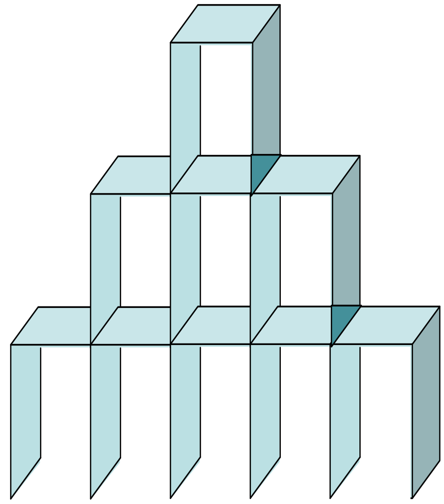

a) Who uses more cards to create a 4 story version of their figure? Justify your answer with mathematical evidence. <u>(3)</u>

b) How many cards would it take Mr. Tran to make his card structure 10 stories tall? How about 200 stores tall? <u>(3)</u>

c) If Mr. Goza builds his structure to a height of 25 stories, how many cards will he need for the bottom level? <u>(2)</u>

d) Create a quadratic function *C(x)* to match the number of cards in Mr. Goza's structure when it is *x* stories tall. <u>(2)</u>

Created and designed Nate Goza and Liem Tran for

Name: Date: Period:

e) Use factoring to find out how many stories Mr. Goza's structure will have if he uses 210 cards to make it. (Hint: (20)(21) = 420) (2)

f) How many more ***stories taller*** will Mr. Tran's structure be than Mr. Goza's if each of them are given 3 decks of cards to make their structure. Assume each deck has 52 cards. (Your answer to # 2 should help! You may want to use the Quadratic Formula for Mr. Goza's structure.) (4)