## Concept 1: Relations and Functions

Common Core Standards: F-IF 1, 2, 4, 5, 6 N-Q 1, 2, 3

Key Vocabulary:
Set
Relation
Function
Graph
Mapping
Ordered Pairs
Table
Input
Independent Variable
Domain
Output
Dependent Variable
Range
Interval
Interval Notation
Set Notation
Function Notation
Vertical Line Test
Discrete
Continuous
Maximum
Minimum
Increasing
Decreasing
Average Rate of Change
Instantaneous Rate of Change
Position Function
Velocity

## 1.1: Sets of Numbers

One or more distinct numbers which have something significant in common make up a set.
To display a set of numbers we can use set notation: $\{a, b, c, \ldots, s\}$ where $a, b, c$ and so on are the elements of the set and $\mathrm{a}<\mathrm{b}<\mathrm{c}<\ldots<\mathrm{s}$.

Example: "The set of the first four positive even integers" is $\{2,4,6,8\}$
Each picture below has numbers in it. Your task is to decide whether those numbers are related in such a way that they form a set.

- If the numbers form a set, use set notation to represent the set mathematically and explain what the set represents.
- If you feel the numbers do not form a set, explain why you came to this conclusion.

Example:


Can you find another set of numbers in this image? What does the set represent?
1)

2)

3)

4) Entrees



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sa3.55
VEAL CHRP WITH SWEET
AND HET FERPERS


 **2. $4 *$
5)

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FIRSTNAME LASTNAME
6)

7)

|  |  |  |  |  | Mar |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 1 | 2 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 21 |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |

8) 


9)

10)

11)

| American League Leaders |  |  | National League Leaders |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| AL. BATTING AVERAGE |  | AVG | NL. BATTING AVERAGE |  | AVG |
| T | 1. Prince Fielder, TEX | . 349 |  | 1. Deee Gordon, MIA | . 356 |
|  | 2. Jasorn Kipnis, CLE | . 333 |  | 2. Paul Goldschmidt, ARI | . 349 |
|  | 3. Nelson Cruz, SEA | . 326 |  | 3. D) Lemahieu, COL | . 348 |
|  | 4. Miguel Cabrera, DET | . 325 |  | 4. Norl Aokl, SF | . 333 |
| Fleider | 5. Mike Moustakas, KC | . 318 | Gordon | 5. Bryce Harper, WSH | . 328 |

## 1.1b: Related Sets

Look at the graphic below from the U.S. Department of Labor.

## Earnings and unemployment rates by educational attainment



Note: Data are for persons age 25 and over, Earnings are for full-time wage and salary workers.
Source: Current Population Survey, U.S. Bureau of Labor Statistics, U.S. Department of Labor

1) List 2 or 3 things you "notice" that you find interesting.
2) What are 2 things you "wonder" about after looking at the information in the graphic.

The college counselor wanted to share this information with the students but she felt that a table would be a clearer representation of the information in the graphic. She made this table:

| Approximate Years of Education <br> (After High School) | Median Yearly Income (\$) |
| :---: | :---: |
| $0^{*}$ | 34,736 |
| 1 | 38,532 |
| 2 | 41,184 |
| 4 | 57,252 |
| 6 | $\mathbf{6 8 , 9 5 2}$ |
| $\mathbf{8}$ | $\mathbf{8 5 , 2 2 8}$ |

* Assuming HS Diploma. Less than HS Diploma has an MYI of \$25,376

Talk with your group about where how the numbers in the table the College Counselor made are related to the numbers from the graphic presented at the beginning of the task.
3) Do you think the counselor's table makes sense? Why or why not?
4) Based on the information in the table and the graphic from the U.S. Department of Labor, do you agree or disagree with the following statement?

There is a relationship between the number of years of education a person receives and the amount of money they can be expected to earn each year.

## Explain your choice.

5) The table the College Counselor made contains two sets of numbers. Use set notation to represent each set.

When two sets are related in a specific way such that every element of one set is related to one or more elements in the other set we call it a relation.
6) Explain why the information in counselor's table can be considered a relation.

## Extension:

On average, how much does one year of Pre-College education affect your future expected yearly income? How did you come to this conclusion?

On average, how much does one year of College education affect your future expected yearly income?

## 1.2: Relations and Functions

When two sets are related in a specific way such that every element of one set is related to one or more elements in the other set we call it a relation.

A relation is can be thought of as a set of paired numbers consisting of input and output values.
The set of input values make up the domain of the relation.

The set of output values make up the range of the relation.
A function is a relation such that every input has only one output.
From the Common Core Standards (F-IF 1):

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.

Each chart below represents a relationship between two sets (a relation). Fill in each chart with the appropriate labels and or numbers. Explain, in writing, why your answers make sense.

Relation 1.

| Pairs of Shoes | 1 | 2 | 3 | 4 |  | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Shoes |  | 4 |  |  | 10 |  |  |

Relation 2.

| Weeks Until the Party | 10 | 8 | 6 |  | 3 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Days Until the Party | 70 |  | 42 | 35 |  | 7 |

Relation 3.

| Hours in the Car on a Road Trip | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 65 | 135 | 200 |  | 325 |

Relation 4.

|  | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 60 | 120 | 180 | 240 |  |

Relation 5.

|  | 1 | 2 |  | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 31 | 28 | 29 | 31 | 30 | 31 | 30 | 31 | 31 | 30 | 31 | 30 | 31 |

Relation 6.

|  | 1 | 2 | 3 | 8 | 20 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 3 | 4 | 5 | 10 |  | 50 |

Relation 7.

| Number of Tickets ( $t$ ) | 1 |  |  | 2 |  |  | 3 |  |  | 4 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Price for $t$ tickets (\$) | uppr | middle | Lowe | uper | middle | Iower | upper | Midde | Iower | upper | ${ }_{c}^{\text {Midd }}$ | Iower |
|  | 10 | 20 | 50 | 20 |  | 100 | 30 | 60 |  |  |  | 200 |

Mr. Tran thinks there are more possible outputs for 2, 3, and 4 in Relation 7. Explain why you agree or disagree (mathematically).

## Follow Up Questions for 1.2

1) Based on your completed charts, which of the 7 relations are functions? Explain how you know.

Every relation has a domain and a range. The domain and range of are sets of numbers.
2) What is the domain of Relation 5 ?
3) What is the range of Relation 1?
4) What is the range of Relation 2?
5) What is a "reasonable domain" for Relation 7. Explain your answer.

## Unit 1 CYO 1: Representing a Relation

Create your own relation from scratch. There have to be at least 5 input and output values. Represent your relation in the same way that the relations were displayed in 1.2.

Explain, in writing, why your relation makes sense.

Think of another way to represent this relation. Clearly display the new representation of your relation in the space below or on a separate sheet of paper. Make sure that the way you represent your relation makes it clear which inputs go with which outputs!

Share your relation with the people in your group. As a group, create a chart you all agree on. Either pick one that someone already has, or make a new one together.

You cannot use a chart or table to represent your relation. You need to come up with a different way to represent the relation you choose. Make sure that the way you represent your relation makes it clear which inputs go with which outputs! Present your new representation on poster paper with a clear explanation of why the relation makes sense.

## 1.3: Representing Relations in Multiple Ways

Four different relations are shown below using different representations. On the paper provided, represent each of the relations with the 3 other ways.

Relation 1 is represented with a "table":

| input | output |
| :---: | :---: |
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 30 |
| 4 | 40 |

Relation 3 is represented with a "mapping":


Relation 2 is represented with a "graph":


Relation 4 is represented with a "set of ordered pairs":
$\{(1,2),(2,3),(2,5),(4,1),(5,5),(5,1),(7,2)\}$

## Relation 1:




## Relation 3:



## Relation 4:



## Follow Up Questions for 1.4

1) Which of the relations in this assignment are functions? Which are not? Explain how you know?
2) Given any relation, how can you tell whether or not it is a function?
3) If you were only given one representation of a relation and were asked questions about the relation, which representation would you prefer to have? Explain why?
4) Extension: Pick one or two of the relations in this task and create a scenario that could match that relation.

## 1.4b: Multiple Representations

1) Show the relation $(-2,2),(-1,-2),(0,2),(1,-2),(2,0)$ with a
a) table
b) mapping
c) graph
d) What is the domain of the relation?
e) What is the range of the relation?
f) Is the relation a function? Why or why not?

## 1.4c: Getting in Shape

Today I plan to run 7 miles. For the next three days I will run two fewer miles each day. Then I'm taking a day off.

1) Represent this relation with a table. 2) ... with a set of pairs.
2) ... with mapping.
4).. with a graph.

3) What is the domain of the relation?
4) What is the range of the relation?
5) Is this relation a function? Why or why not?

## 1.4: Mini Mart Madness aka A Mountain Doozey

A local Mini-Mart sells sodas in different ways:
Individual Sodas \$1
Six-Packs \$4
Twelve-Packs \$7


Does the picture match the scenario? Why or why not?

We might all assume that everyone's choice is to buy sodas in the cheapest way possible, but who knows. Check out these people:
https://youtu.be/1017dc47t40


## 1.4: Presentation Questions

Considerations:

- What if the buyer doesn't care about how much the sodas will cost?
- Are there multiple options for people who don't know/care that there is a cheapest way, or don't plan ahead to save money?

The relationship between the number of sodas you want to buy " $S$ " and the cost of buying those sodas " $C$ " is a relation. Explore this relation as thoroughly as possible for up to 15 sodas.

Also explore the relationship between the number of sodas you want to buy " $S$ " the least expensive way to buy those sodas " $L$ ".

Discuss and justify whether or not these two relations are functions.

## 1.4: Scaffolding Questions

1) How much will it cost you to buy 2 sodas?
2) How much will it cost to buy 7 sodas?
3) What is the cheapest way to buy 10 sodas?
4) I gave my nephew $\$ 10$ and sent him to get 8 sodas for him and his friends. He came back with the sodas and $\$ 2$ change. What did I have to explain to him?
5) 13 sodas can be bought 4 different ways. What are they and how much does each way cost?
6) Think about the relation with input 0 to 15 sodas and output which is the cost of buying those sodas.
a) Display this relation with a table or a mapping.
b) Display this relation on the graph below

c) Is this relation a function? Why or why not?
7) Now think the relation with input 0 to 15 sodas and output which is the cost of the cheapest way to buy that number of sodas.
a) Display this relation with a table or a mapping.
b) Display this relation with a Graph.

c) What is the maximum value of this relation? What does it represent?
d) Is this relation a function? Why or why not?

## 1.4: Follow Up Questions

1) Can you think of a scenario where it would make sense that someone would buy 13 sodas by purchasing one six pack and 7 individual sodas.
2) Mr. Tran argues that 14 sodas can be bought in more ways than Mr. Goza thinks is possible. Tran says that you can buy 14 sodas by buying a twelve pack and 4 individual sodas which only costs $\$ 11$. Mr. Goza says that shouldn't count because no one would ever buy 16 sodas to get 14 sodas. Tran asks, "Well then why do you think buying a 12pack to get 10 sodas counts?" Do you agree with Tran or Goza? Which possibilities should count for this relation?
3) Predict how many different ways you could buy 30 sodas. Is there a short way to find out the number of possibilities there are? Is there a pattern?
4) What is the cheapest way to buy 30 sodas? What is the cheapest way to buy $S$ sodas?
5) The mini-mart is considering selling 24 packs in the future. How much do you think they will charge considering how they have priced their other sodas? How did you decide on this price?
6) Can you come up with another scenario that can be described by a relation that is not a function? Explain.

## 1.4b: The Great Shirt Sale

The Banana Republic Outlet sells plain T-shirts for \$7. Consider the relation with input equal to the number of T-shirts bought and the output is the cost of buying the given number of T-shirts from Banana Republic.

1) Create a table to represent this relation for buying 0 to 10 shirts.
2) What is the range of this relation?
3) Represent this relation with a graph.

4) Is this relation a function? Why or why not?

Meanwhile, next door, Express has a sale on designer shirts. All graphic T-shirts are \$10, polo shirts are $\$ 20$ and button-up shirts are $\$ 30$.
5) How much will 1 shirt cost?
6) How many ways can you buy 2 shirts? List them.
7) How much will 2 shirts cost? Consider all possibilities.

Imagine that we attempt to make the same kind of relation as we did in Numbers 1-4. Then the input would be the number of shirts bought and the output would be the cost of buying that number of shirts from Express.
8) Sketch a graph to represent this relation for buying 0 to 3 shirts.

9) Is this relation a function? Why or why not.

## 1.5: Seven Somethings

Check out the chart below. It represents a real-life relation.

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 21 | 29 | 33 | 38 | 26 | 23 |

Represent this relation with ...

1) a Mapping
2) a table
3) a Graph

4) Is this relation a function?

Why or why not?
5) Make up a story/scenario to match this set of data. Fill in the blanks

The table on the front of the page actually represents the number of points Kobe Bryant scored in each game of the 2010 NBA Finals. In the NBA Finals two teams play each other in a "seven game series" until one team wins 4 games.
6) Explain why there are seven numbers in the domain of this function.
7) What is the maximum value of the function? What does this value represent? What does this imply about the scenario?
8) If the situation had gone differently, what is the minimum number of values that could have been in the range. How could this have been possible?
9) Explain why this scenario guarantees that this relation would be a function.
10) Is this also true for the scenario you created in \#5? Explain.

## Quiz \#1: Functions

1. Over the last 5 school nights, how many minutes per night did you spend on your homework? Fill in the chart with the information.

|  | 1 | 2 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

2. Represent this relation with a...
a) table
b) mapping
c) set of ordered pairs
d) graph

3. What is the domain of the relation?
4. What is the range of the relation?
5. What is the minimum value of the relation? What does this value represent?
6. Is this relation a function? Why or why not?
7. Is the relation continuous or discrete? Explain how you know?
8. Draw a mapping of a relation that is not a function.
9. Draw a possible graph of a function that is continuous with a domain $[0,10]$.

## Quiz \#1: Functions (Version B)

1. Over the last 5 nights how many hours per night did you spend on your homework? Fill in the chart with the information.

|  | 1 | 2 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

2. Represent this relation with a...
a) table
b) mapping
c) set of ordered pairs
d) graph

3. What is the domain of the relation?
4. What is the range of the relation?
5. What is the minimum value of the relation? What does this value represent?
6. Is this relation a function? Why or why not?
7. Is the relation continuous or discrete? Explain how you know?
8. Draw a mapping of a relation that is not a function.
9. Draw a possible graph of a function that is continuous with a domain $[0,10]$.

## 1.6: Intervals and Interval Notation

An interval is a set of numbers with the property that any number in between two numbers in the set is also in the set.

For example all the numbers between 0 and 3 form an interval.

* List 4 numbers in that interval:

Intervals can be displayed with number lines, inequalities, or interval notation. Example:


In the example, 2 and 6 are endpoints of the interval.
We use [, ] if the endpoints are included and (, ) if they aren't.
Or a combination if only one endpoint is included. Check out the example.

* How would you write "all the numbers between 0 and 3 " with interval notation?
* How would "all the numbers up to 10 starting at zero" look in interval notation?

For each scenario decide if the numbers form an interval. Write the set using either set notation or interval notation depending on which is appropriate. Be ready to argue your choice.

1) The low temperature tomorrow will be 45 degrees and the high temperature will be 72 .
2) Paul Blart will eat 1 to 5 donuts this morning.
3) My next tweet will get up to 10 favorites.
4) An NFL player can run the 40 -yard dash in 4.2 to 7.5 seconds.
5) I drink somewhere between 12 and 20 ounces of coffee on most days.
6) When I roll one of the dice, which table could you end up at?
7) My wife is in her thirties.
8) A golf ball that started on the ground went 50 feet up into the air.
9) A car accelerated from 0 to 100 real quick.

## 1.7: Continuous vs. Discrete

As Mr. Lough gets in shape he is able to run more and more each week. He is also able to get more consistent with his pace. During his first week back coaching the cross country team he is able to run 2 miles. Each week he runs 2 more miles than he did the week before. He continues this pattern for 5 weeks.

Complete the "mapping" below and then create a graph to match the function in the scenario.


What is the domain of this function?


What is the range of this function?

What is the maximum value of this function?

Estimate the number of miles Mr. Lough will run on the $7^{\text {th }}$ week if he continued to run.

To give him a break from running, Mr. Bystrom takes Mr. Lough hiking. Mr. Bystrom has an apple watch to keep track of how far he has walked each time he goes out. The two choose a 10 mile hike. The hike is a loop trail and Bystrom and Lough complete the whole hike in five hours keeping a constant pace the entire time.

Complete the "table" below and then create a graph to match the function in the scenario.

| Time <br> (hrs) | Distance <br> (miles hiked) |
| :---: | :---: |
| 0 | 0 |
| 1 |  |
| 2 | 4 |
| 3 |  |
| 4.5 |  |
|  | 10 |



What is the domain of this function?

What is the range of this function?

What is the maximum value of this function?

What do the two functions have in common? How are they different?

## 1.7b: Scenarios, Functions, and Graphs

1) I drink water by filling my water bottle and refilling it throughout the day.

| Day of the week | Fluid ounces of <br> water consumed |
| :---: | :---: |
| 1 (Monday) | 40 |
| 2 | 27 |
| 3 | 60 |
| 4 | 20 |
| 5 | 40 |

a) Is the relation a function? Why or why not?
b) Is this relation discrete or continuous? How do you know?
c) Represent the relation with a mapping.
d) Represent the relation with a graph. Be as accurate as possible.
e) How many fluid ounces of water does my bottle most likely hold? How did you come to this conclusion?
2) The graph below shows Mr. Goza's distance from home throughout the day, starting from 10:00 AM when he left his house.

a) Describe the domain and range of the graph using numbers and units. (Hint: Use intervals!)
b) At what time did Mr. Goza reach his furthest destination? How far from home was he at that time?
c) Does this graph represent a function? How do you know?
d) At what times was Mr. Goza at home?
e) At what times was Mr. Goza 60 miles away from home?
3) On the plane below, draw a graph to match the given scenario. Make up units and numbers for the horizontal and vertical axis.

The amount of violent crimes in our neighborhood was increasing for months. Then during the summer Mr. Goza moved in and the number of crimes decreased dramatically. We suspect that he is a Superhero.

a) What is the domain?
b) What is the range?
c) What is the maximum value of the function? What does it represent?
4) On the plane below draw a relation that is not a function. Explain how you know.

5) Show the relation $(-2,-8),(-1,-1),(0,0),(1,1),(2,8)$ with a $\ldots$
a) mapping
b) graph

c) What is the domain of the relation?
d) What is the range of the relation?

## 1.8: Connecting the Dots (or Not!)

Each table below represents a real life relation. Plot the points from the table into the graph next to it. Then decide whether or not you should connect the points depending on the scenario. Explain your decision. If you should connect the points, connect them in any way that makes sense to you (and matches the scenario).

1) The table shows the number of home runs a baseball player had in his first four seasons in the Major Leagues.

| Seasons <br> played | Home <br> Runs Hit |
| :---: | :---: |
| 1 | 27 |
| 2 | 30 |
| 3 | 38 |
| 4 | 44 |
| 5 | 29 |

Domain:


Range:
2) The table shows the miles you have traveled in a car on a trip out of town for 5 hours after you left your home.

| Hours since <br> you left home | Miles away <br> from home |
| :---: | :---: |
| 1 | 60 |
| 2 | 125 |
| 3 | 200 |
| 4 | 210 |
| 5 | 210 |

Domain:


Range:
3) The table shows the number Facebook posts my younger cousin made over the last 8 months (from this year).

| Month | Facebook <br> Posts |
| :---: | :---: |
| 1 | 22 |
| 2 | 40 |
| 3 | 33 |
| 4 | 0 |
| 5 | 18 |
| 6 | 50 |
| 7 | 44 |
| 8 | 30 |

Domain:


Range:
4) The table shows the height of a hot air balloon that takes a couple on a romantic ride over the Grand Canyon.

| Minutes since <br> takeoff | Height (feet) |
| :---: | :---: |
| 10 | 100 |
| 20 | 300 |
| 30 | 450 |
| 40 | 450 |
| 50 | 200 |
| 60 | 0 |

Domain:


Range:

If a function contains intervals where all points are connected in such a way that there are no jumps, breaks, or gaps in its graph is said to be continuous on those intervals. A function that is continuous over its entire domain is called a continuous function.
5) Which of the functions in numbers 1-4 are continuous functions?

## 1.9: Creating Graphs to Model a Scenario

Use the following information to create graphs on the graph paper attached. Make sure that your horizontal axis is always represents some unit of time. Any information that is not given can be made up by you! Be creative!

1) The height above ground level of a man who enters the elevator on the six floor, rides to the underground parking lot to get his laptop from his car and returns to the fourth floor for a meeting.
2) The number of people at the Staples Center starting two hours before a Lakers game and ending two hours after the game.
3) The temperature outside on a winter day in the mountains.
4) The height above sea-level of a piece of bait on a deep sea fisherman's line as he fishes in the Pacific Ocean (from baiting the hook to catching the fish).
5) The approximate number miles you walked/ran on each of the last ten days.
6) The speed of a soccer ball that has been kicked straight up in the air, starting the moment it is kicked and ending the moment it hits the ground.

For each problem sketch a graph that matches the scenario described. Next to each graph describe why your graph looks the way it does. Try to use the words increasing, decreasing, and constant. Also, estimate the domain and range of your graph. Make sure to label your axis with both units and numbers.




Domain:
Range:
3) Explanation:

Domain:
Range:



4) Explanation:

Domain: Range:
5) Explanation:

Domain:
Range:
6) Explanation:

Domain:
Range:

## 1.9b: More Graph Stories

Graph each scenario on a piece of graph paper. Include domain and range, name your functions, and provide equations for the graphs (ex: height $=f($ time $)$ ).

1) The heart rate, in beats per minute, of a runner from 5 minutes before she starts her run until a half hour after her run is complete.
2) The approximate number of text messages you sent in each month of this year.
3) The height (in feet of elevation) of a snowboarder from the time he gets in line for the ski lift to the time he gets back to the bottom of the mountain.
4) The distance from home of a person who has gone out for the day. (You make up the details of the story.)
5) The height above Earth of a UFO that comes down from outer-space, hovers above Earth just long enough to abduct Kanye West, and takes off back to another galaxy.

### 1.10: Double Mata-thon

Mr. Mata is running a double marathon. The table below shows the number of miles Mr. Mata has run after $t$ hours is shown in the table below.

| $t$ <br> (hours since the race <br> began) | TotalNumber of Miles <br> Completed <br> 1 |
| :---: | :---: |
| 2.5 | 8 |
| 3 | 19 |
| 4 | 23 |
| 4.5 | 30 |
| 5 | 33 |
| 6 | 33 |
| 8 | 41 |
| 9 | 51 |
| 10 | 52.4 |
|  | 52.4 |

1) After 3 hours how far has Mr. Mata run?
2) After 6 hours, how many miles has Mr. Mata run?
3) How long does it take for Mr. Mata to run the first 30 miles?
4) What was Mr. Mata's average speed between the $6^{\text {th }}$ and $8^{\text {th }}$ hour?
5) Mr. Mata got injured along the way. How many hours into the race did this occur? What evidence from the table led you to your answer?
6) On the plane below, graph the data from the table. Think about whether or not it makes sense to connect the points.

| $t$ <br> (hours since the <br> runner began) | Miles <br> Completed |
| :---: | :---: |
| 1 | 8 |
| 2.5 | 19 |
| 3 | 23 |
| 4 | 30 |
| 4.5 | 33 |
| 5 | 33 |
| 6 | 41 |
| 8 | 51 |
| 9 | 52.4 |
| 10 | 52.4 |


7) Use your graph to estimate how far Mr. Mata has run after 2 hours.
8) Fill in the blanks:

After 8 hours Mr. Mata has run $\qquad$ miles.

After 5.5 hours Mr. Mata has run approximately $\qquad$ miles.
9) Come up with 2 more statements like the ones in Question 7 using your graph to approximate the distance at times of your choice.

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.

If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y=f(x)$.
$f$ is the most common way to "name a function." Probably because the word function starts with an f . However you can name a function in a variety of ways (usually we use a single letter). For example, we can call the function describing Mr. Mata's distance with an $M$. Since we have been using the input variable $t$ the output of our function can be written as $M(t)$. This allows us to write about the function in a quick, easy way.

We can now say $M(8)=51$. This means "After 8 hours, Mr. Mat has run 51 miles."
10) Write an equation to match the statement: "After four and a half hours Mr. Data has run 33 miles."
11) Write a statement to match the equation $M(9)=52.4$.
12) $M(6)=$ $\qquad$ What does this equation tell us?
13) Use your graph to estimate $M(7)$. What does your answer represent?
14) What is $M(0)$ ? How do you know this?
15) Create two more equations like the one in \#11 based on the data from Mr. Mata's Marathon.
16) How many miles are there in a double marathon? How do you know?
17) $\quad$ Say $M(t)=23$. Then $t=$ $\qquad$ . What does your answer tell us?
18) How long did it take Mr. Mata to finish the double Marathon? How do you know?

### 1.11: Dwayne Wade Signals Hello from a Distance

The following table is shows data of an unknown relation:

| $h$ | $T$ |
| :---: | :---: |
| 0 | 100 |
| 1 | 104 |
| 3 | 104 |
| 5 | 102 |
| 8 | 97 |
| 10 | 92 |
| 12 | 85 |

1) Represent the relation with a mapping.
2) Represent the relation with a graph.

3) Is this a function? Why or why not?
4) What is the domain?
5) What is the range?

We can now say that $T$ is a function of $h$. Then we can say that $T=f(h)$. We can say the input is $h$ and the output is $f(h)$.
6) Is $f$ discrete or continuous? How do you know?
7) What is the maximum value of $f$ ? How do you know?
8) Fill in the blanks:

$$
f(1)=\ldots \quad f(10)=\ldots \quad f(\ldots \quad)=102
$$

The average rate of change of a function over an interval is the amount of change in the output divided by the length of the interval (the change in input).

The average rate of change of the function $f$ on the interval $[\mathrm{a}, \mathrm{b}]$ is:

$$
\frac{f(b)-f(a)}{b-a}
$$

where $f(b)$ and $f(a)$ are (as always) the outputs of $b$ and $a$.
9) Calculate the average rate of change of $f$ on the interval $[0,3]$.
10) Calculate the average rate of change of $f$ on the interval $[8,12]$.

The reason that this assignment was called "Dwayne Wade Signals Hello from a Distance" is because the data in the table represents the temperature in Miami starting at 12:00 PM (noon) and ending at 12:00 AM (midnight) during a Heat Wave. (Get it? Miami Heat Wave?). Here is a picture of the actual weather from that day.


Think about and discuss with your group: How is the data from the table on the first page consistent with the temperatures shown in the weather report above?
11) Knowing that $f$ represents the temperature in Miami throughout a given day changes our answers from \#2 - \#6 significantly and gives us insight to what our answers mean. With a colored pen or pencil go back and re-answer 2-6. Don't erase your old answers!
12) Write an equation to match the function now that you know what it is about. Do you still want to call the function $f$ ?
13) Does the new information about $f$ change our answer to \#7? Why or why not?
14) Now that we know $f$ is about the temperature in Miami, explain what your answer to \#8 means in terms of the situation.
15) Explain why your answer to \#9 is negative based on the situation.
16) What is the average rate of change of $f$ on the interval $[0,12]$ ? Why is this answer deceiving/confusing?
17) Explain why any relation where the input is time and the output is an outdoor temperature is going to be a function.
18) Explain why any function where the input is time and the output is an outdoor temperature is going to be continuous.

### 1.12 Functions in the LBC

I recently took a trip down to Long Beach to see Jerry Seinfeld perform stand-up comedy. From the time I left my house at 5:00 PM, I kept data on how far I was from my home (shown in the table below). On the way there I picked my wife up from work, we got some drive-thru for dinner, fought some traffic and then got lost trying to park. The show was very funny so it made the trip worthwhile.

| Hours <br> since <br> 5:00PM | Mlestrom <br> home |
| :---: | :---: |
| 0 | 0 |
| 0.5 | 10 |
| 1 | 15 |
| 2 | 30 |
| 2.5 | 35 |
| 4 | 35 |
| 4.5 | 35 |
| 5 | 0 |

Let's think about the function $L$ that will output my distance from home when a given number of hours since 5:00 PM is input.

Position in the real world is measured in units of distance. An object's position is its relative distance from some fixed point. For example if you can jump 35 inches high, the maximum position during one jump is 35 inches. In this case distance is relative to the ground.

A position function represents the position of an object at given time $t$ for every time in its domain.

1) True or False: $L$ is a position function. Explain your answer.
2) True or False: $L$ is a continuous function. Explain your answer.
3) How far was I from home at 5:30 PM. How do you know?
4) What is $L(2)$. What does your answer represent?
5) Estimate $L(3)$. Justify your estimation.
6) How long do you think the comedy show was? What evidence do you have for this?
7) About how many miles is it from my house to Long Beach? How do you know?
8) Sketch a possible graph of Distance $=L(h)$ given what we know about the function.

9) What is the domain of $L$ ?
10) What is the range of $L$ ?
11) On what interval(s) is $L$ increasing? How does that match my story?
12) On what interval(s) is $L$ constant? How does that match my story?
13) On what interval(s) is $L$ decreasing? How does that match my story?

The rate of change of position is called velocity.
14) What was my average velocity on the way to Long Beach? Use $\frac{f(b)-f(a)}{b-a}$.
15) What was my average velocity on the way home?
16) Explain why the answer from \#16 is negative.

For any function $f$ we say that a number $c$ such that $f(c)=0$ is a root of $f$. In other words any input that outputs 0 is a root of $f$. We also say that $c$ is the $\boldsymbol{x}$-intercept of the graph of $y=f(x)$.
17) The function $L$ has two roots. What are they?
18) What do the roots tell us about Mr. Goza in this problem?

### 1.13: Equations and Functions

The graph below represents a hiker's distance (in miles) from the trailhead $t$ hours after starting a hike.


1. Mr. Goza decided to call this function $f$. Why do you think he chose the variable $t$ ?
2. Did Mr. Goza have to use $f$ ? Explain why or why not? Would you choose another name?
3. What is $f(3)$ ? What does it represent?
4. There is one solution to the equation $f(t)=10$. What is that solution and what does it represent?
5. There are two solutions to the equation $f(t)=2$. What are the solutions and what do they represent?
6. Explain why there are an infinite number of solutions to the equation $f(t)=4$.

For any function $f$ we say that a number $c$ such that $f(c)=0$ is a root of $f$. In other words any input that outputs 0 is a root of $f$. We also say that $c$ is the $x$-intercept of the graph of $y=f(x)$.
7. What are the roots of this function?
8. What do the roots represent in this scenario?
9. Write a general equation to represent this function.

### 1.13b: More Functions Practice



1) Is the relation shown in the graph a function? Why or why not?
2) How many roots does this function have? What are they?
3) Is the relation continuous or discrete? How do you know?
4) What is the domain of the relation? 5) What is the range of the relation?

Assume that the function $S$ graphed below represents a runner's speed during a 100 meter sprint from the time the race starts to the time she comes to a rest at the end.


1) Is this function continuous or discrete? How do you know?
2) What is $S(2)$ ? What does your answer tell us?
3) What is the domain of this function?
4) How many seconds do you think it took for the runner to complete the race? How did you come to this conclusion? (Your answer should not be 18 seconds!)
5) What is the maximum value of the function? What does it represent?
6) What are the two roots of $S$ ? What do they represent?

### 1.14: Mad-Lib Madness

The $\qquad$ in $\qquad$ of a $\qquad$ changes continuously as a function of time.

This occurs over a 10 $\qquad$ time interval.

The maximum $\qquad$ of the $\qquad$ is 50 and the minimum is zero.

Create the graph to match your story you created above on the interval $[0,10]$.
The graph below is $\qquad$ $=$ $\qquad$ (t).


Why does your graph look the way it does? Provide details of the story that explain the graph.

Use your graph to answer the following questions. Explain what your answers represent in terms of the scenario.

1) Complete the equation: $f(5)=$ $\qquad$
2) Find the solution or solutions to the equation $f(t)=30$
3) Complete the equation: $f(0)=$ $\qquad$
4) Find the solution or solutions to the equation $f(t)=0$

### 1.15: Advanced Graph Stories

For each of the following scenarios create the graph of a function with the given input and output to match the situation that is described. Include domain, range, and an equation for each of the functions you graph.

1) I walked to a friend's house that was 6 miles away. On the way my average velocity was 2 mph . I stayed for 4 hours. On my way home I was in a bit of a hurry so I walked faster (with an average velocity of -4 miles per hour).
Input: time
Output: distance from my house


Equation:
2) Starting from the top of a 12,000 foot high mountain a snowboarder rides down 3,000 feet. He stops there to catch a lift to the top of another mountain that is even higher than the one he just rode. He waits 5 minutes in line at the lift before getting on. This lift is slow and it takes a while to get to the top of the mountain. Input: time (since he started down the first mountain)

Output: Elevation


Explanation:

Equation:
3) Over the last ten days I've been trying to run more. I've only taken 3 days off. The most miles I ran in one day was 7 , the least (besides days off) was 2 . I would typically take days off after longer runs.
Input: days Output: miles ran


Equation:
4) A scuba diver jumps from his boat into the ocean. He proceeds to dive 100 feet below sea level in 2 minutes. He stays there looking at an eel for 2 minutes then dives deeper. After 15 minutes he heads back up to the boat at a quickened pace because he fears that there are sharks nearby.
Input: time Output: height above sea level


Domain:
Range:
Equation:

## Unit 1 CYO 2: Create Your Own Function Graph

In class we did a set of problems where you were asked to draw a graph to match a situation that was described on the worksheet. Now I am going to ask you to make up your own situation and draw a graph to go with it.

Requirements (you will receive a zero if these are not met)

- Assignment must be done on poster size paper.
- Your poster must include a graph with a description of the situation/phenomena/story being represented.
- Your poster must be turned in by $\qquad$ .
- You may work individually or with one partner of your choice.


## Rubric

I will be checking to see if:

- The curve you draw accurately matches the situation/phenomena/story you describe. The graph is clear and the description is well written. >> 10 points
- Your units and labels for your axis are clear, correctly scaled, numbered, and make sense according to your description. >> $\mathbf{3}$ points
- The situation you describe applies to a real life or fictitious situation that actually makes sense. >> 2 points
- The subject matter of your poster is creative and/or interesting. >> $\mathbf{3}$ points
- The poster looks presentable and all work/thought is clearly shown. >> $\mathbf{3}$ points
- The domain and range is included and correctly given. >> 2 points
- Your function is named and the equation for the graph is given. >> 2 points

I look forward to laminating some of these for permanent display. Extra credit will be given to posters that get laminated. There is a lot of freedom here. Be creative and make them look good!

This project will be worth $\mathbf{2 5}$ points and will go into your tasks category.

### 1.16: Vocabulary Practice

Easy: On a separate sheet of paper create a possible example of each:

1) A mapping of a relation that is not a function.
2) A set of ordered pairs of a relation that is a function.
3) A discrete graph of a relation that is not a function.
4) A graph of a relation that is a continuous function.
5) A table of a relation that is a function.

Harder: For each problem 6-12 create a graph for a function ...
6) with a maximum value 20 and a minimum value of 5 .
7) with a domain $[0,6]$ and a range $[-5,4]$.
8) where the average rate of change on $[0,5]$ is 2 .
9) that is discrete with a story to match.
10) that is always decreasing with a story to match.
11) made up of 4 lines with different Rates of Change.
12) that is constant on the interval [5,9] with a story to match.

### 1.17: The Freaky Falcon Function

The function $F$ approximates the height (in meters) of a falcon $t$ seconds after he takes flight from a tree branch. The graph of the equation height $=F(t)$ is shown below.


1. Appropriately label the axis in the graph above.
2. What is the domain of $F$ ?
3. What is the maximum value of $F$ ? What does it represent?
4. What is $F(6)$ ? What does you answer tell us?
5. How many roots does $F$ have? What are they? What do they represent?
6. On what interval(s) is $F$ increasing? What does this tell us about the falcon?
7. Is $F$ continuous or discrete? How do you know?
8. What are the solutions to the equation $F(t)=8$ ? What do your answers represent?
9. How high is the tree branch that the falcon is perched on before he takes flight? How do you know?

## Unit 1 Test Review

Mr. Beltran and his wife want to build a pool in their backyard.
They decide to drive out to the middle of the Mojave Desert to pick up some Minions to help them.


The function $B(t)$ graphed below shows the Beltran's distance from home (in miles) during their trip to pick up the minions, where $t$ is the number of hours that have passed since they left.


1. What is the domain of $B$ ?
2. What is the maximum value of $B$ ? What does it tell us about the Beltran's trip?
3. What are the roots of $B$ ? What do they represent?
4. On which interval is the function decreasing? What does that tell us about the Beltran's?
5. When they return home Mrs. Beltran complains that there is too much traffic near where they live. Identify some evidence from the graph that supports this claim.
6. What is $B(8)$ ? What does the answer represent?
7. What was the average rate of change of $B$ on the interval [0,3]? What does your answer represent?
8. What are the solutions to the equation $B(t)=80$. What do your answers represent?
9. At what time were the Beltran's traveling the fastest. Explain how you know.
10. How long did the Beltrans spend in the desert picking up the Minons? What graphical evidence is there to support your answer?

## Unit 1 Exam: Functions and Relations

## PART 1: Consider the following...

This summer I spent 3 weeks in Park City, Utah which has a great system of bike paths. I decided to ride one of the paths to a lake. The lake was ten miles away on the path from the hotel where I was staying. I left at 12:00 PM and planned to return by 4:00 PM to get ready to go out for dinner with some friends.

Assume $\boldsymbol{f}$ is a position function that measures my distance from the hotel (in miles) from the time I left to the time I returned where $\boldsymbol{t}$ is measured in hours since 12:00 $\mathbf{~ P M}$.

Sketch one possible graph of $f(t)$ to represent my trip to the lake on the axis shown below. Provide a detailed story to match your graph. Then answer the ten questions on the following page. Since you will be using your graph to answer the questions it is recommended that you read the ten questions before creating your graph and story. (7 points possible)


Story:

Use your graph to answer the following questions:

1) What is the domain $f$ ? (2)
2) What is the range? (2)
3) According to your graph, at what time did I get to the lake? (2)
4) According to your graph, what is $f(1)$ ? What does this answer represent? (2)
5) According to your graph what was my average velocity on the way to the lake?
6) According to your graph, on which interval(s) was $f$ decreasing? How does that match your story? (2)
7) Is your version of $f$ continuous or discrete? Explain why. (2)
8) What was the maximum value of $f$. What does that represent in the story? (2)
9) We know $f$ is a function. Explain why this is obvious based on what it represents in the real world. (2)
10) For each part of this problem you will get one point for an answer that makes sense, one point for a justification of your answer, and one point for correctly making use of one of the vocabulary words listed below in your justification:

| domain | constant | maximum |
| :--- | :--- | :--- |
| range | velocity | minimum |

How would (or why wouldn't) your graph of $D=f(t)$ change if I told you...
a) I went past the lake to see a waterfall further down the path? (3)
b) I stayed at the lake too long and got back late after hurrying back? (3)
c) I went swimming in the lake? (3)

## Unit 1 Exam Part 2: Complete the graph and the table using the given information.

In Part 1 of this Task you were asked to make up information about my ride to the lake. However, the real details of that ride had not been revealed. Since it was a while back I can't remember all the details either, but here's what I was able to remember:

- My average velocity during the first 30 minutes was 10 miles per hour.
- 45 minutes after I left I stopped for 5 minutes to take pictures of some wildlife.
- At 1:00 I was 8 miles along the bike path ( 2 miles from the lake).
- I arrived at the lake at 1:30.
- I spent one hour at the lake.
- I never went further than 10 miles from my hotel.
- From 2:30 to 3:30 I was riding back with an average velocity of 8 miles per hour.
- At 3:00 exactly I was riding faster than I had all day (around 15 mph ).
- 3.5 hours after I left I stopped to rest for 15 minutes.

| Minutes <br> (after 12:00) | Miles <br> (from hotel) |
| :---: | :---: |
| 0 |  |
| 30 |  |
| 60 |  |
| 90 |  |
| 150 |  |
| 210 |  |
| 225 |  |
| 240 |  |

- I got back to the hotel right on time.



## Unit 1 Exam Part 3: Answer the questions based on the scenario.

Last weekend Mr. Goza drove to San Clemente to meet his cousins for lunch. He fought some moderate traffic on the way. Traffic was heavier on the way home so he stopped in Seal Beach to watch the UCLA Football game with Mr. Tran. After a great Bruin victory Mr. Goza was back on the road. There was still a decent amount of traffic on the way home so it took an hour to get back to LA. The graph $D=G(t)$ below shows Mr. Goza's distance from home (in miles) $t$ hours after he left his house (at 12:00 PM).


1) Correctly label the axis of the graph above. (1)
2) What is the domain of $G$ ? (1) 3) What is the range of $G$ ? (1)
3) What is $G(10)$ ? What does your answer represent? (2)
4) On which intervals is the graph decreasing? What does this tell us about Mr. Goza? (3)
5) What is the average rate of change of $G$ on the interval [0,2]? What does your answer represent? (2)
6) How long did Mr. Goza spend watching the game with Mr. Tran? What evidence from the graph led you to this conclusion? (2)
7) What is the maximum value of $G$ ? What does it tell us about Mr. Goza's trip? (2)
8) What are the solutions to the equation $G(t)=60$ ? What do the answers represent? (3)
9) What are the roots of $G$ ? What do they tell us about Mr. Goza? (3)

## Unit 1 Exam Part 3: Answer the questions based on the scenario.

At Mr. G's Donut Shop single donuts are $\$ 1$ and a dozen (12) donuts cost $\$ 8$.
11) How much would it cost for 5 donuts at Mr. G's? (1)
12) How much would it cost for 25 donuts? (2)
13) Your teacher doesn't know about the special price for a dozen donuts and gives you $\$ 40$ to get 40 donuts for your class. She tells you if there is any change you can keep it for being so kind and picking up donuts for the class. How much money should you end up keeping? (2)
14) Assume you are going to buy 22 donuts.
a) What is the cheapest way to buy 22 donuts? How much will it cost? (2)
b) List all the different ways you could buy 22 donuts and the cost of each way. (3)
15) Complete the graph for purchasing 0 to 24 donuts from Mr. G's. Consider all possible ways to buy them in your graph. (3)

16) Is the relation in the graph a function? Explain why or why not. (2)

