# Unit 1: Relations and Functions 

Common Core Standards: F-IF 1, F-IF 2, F-IF 4, F-IF 5, F-IF 6

Key Vocabulary:
Relation
Function
Graph
Mapping
Ordered Pairs
Table
Input
Independent Variable
Domain
Output
Dependent Variable
Range
Intervals
Interval Notation
Set Notation
Function Notation
Vertical Line Test
Discrete
Continuous
Maximum
Minimum
Increasing
Decreasing
Average Rate of Change
*Instantaneous Rate of Change
Position Function
Velocity

## 1.1: Multiple Representations

Show the relation $(-2,2),(-1,-2),(0,2),(1,-2),(2,0)$ with a
a) table
b) mapping
c) graph
d) What is the domain of the relation?
e) What is the range of the relation?
f) Is the relation a function? Why or why not?

## 1.2: Getting in Shape

Today I plan to run 7 miles. For the next three days I will run two fewer miles each day. Then I'm taking a day off.

1) Represent this relation with a table. 2) ... with a set of pairs.
2) 

... with mapping.
4) ... with a graph.

5) What is the domain of the relation?
6) What is the range of the relation?
7) Is this relation a function? Why or why not?

## 1.3: Mini Mart Madness aka A Mountain Doozey

Your local Mini-Mart sells sodas in different ways:

## Individual Sodas \$1

Six-Pack \$4
Twelve-Pack \$7

1) How much will it cost you to buy 2 sodas?
2) How much will it cost to buy 7 sodas?
3) What is the cheapest way to buy 10 sodas?
4) I gave my nephew $\$ 10$ and sent him to get 8 sodas for him and his friends. He came back with the sodas and $\$ 2$ change. What did I have to explain to him?
5) 13 sodas can be bought 4 different ways. What are they and how much does each way cost?
6) Think about the relation with input 0 to 13 sodas and output which is the cost of buying those sodas.
a) Display this relation with a mapping.
b) Display this relation on the graph below

c) Is this relation a function? Why or why not?
7) Now think the relation with input 0 to 15 sodas and output which is the cost of the cheapest way to buy that number of sodas.
a) Display this relation with a table.
b) Display this relation with a Graph.

c) What is the maximum value of this relation? What does it represent?
d) Is this relation a function? Why or why not?

## 1.3b: The Great Shirt Sale

The Banana Republic Outlet sells plain T-shirts for $\$ 7$. Consider the relation with input equal to the number of T -shirts bought and the output is the cost of buying the given number of T-shirts from Banana Republic.

1) Create a table to represent this relation for buying 0 to 10 shirts.
2) What is the range of this relation?
3) Represent this relation with a graph.

4) Is this relation a function? Why or why not?

Meanwhile, next door, Express has a sale on designer shirts. All graphic T-shirts are \$10, polo shirts are $\$ 20$ and button-up shirts are $\$ 30$.
5) How much will 1 shirt cost?
6) How many ways can you buy 2 shirts? List them.
7) How much will 2 shirts cost? Consider all possibilities.

Imagine that we attempt to make the same kind of relation as we did in Numbers 1-4. Then the input would be the number of shirts bought and the output would be the cost of buying that number of shirts from Express.
8) Sketch a graph to represent this relation for buying 0 to 3 shirts.

9) Is this relation a function? Why or why not.

## 1.4: Drink More Water!

I drink water by filling my water bottle and refilling it throughout the day.

| Day of the week | Fluid ounces of <br> water consumed |
| :---: | :---: |
| 1 (Monday) | 40 |
| 2 | 27 |
| 3 | 60 |
| 4 | 20 |
| 5 | 40 |

a) Is the relation a function? Why or why not?
b) Is this relation discrete or continuous? How do you know
c) Represent the relation with a mapping.
d) Represent the relation with a graph. Be as accurate as possible.
e) How many fluid ounces of water does my bottle most likely hold? How did you come to this conclusion?

## 1.5: Seven Somethings

Check out the chart below. It represents a real-life relation.

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 30 | 21 | 29 | 33 | 38 | 26 | 23 |

Represent this relation with ...

1) a Mapping
2) a table
3) a Graph

4) Make up a story/scenario to match this set of data. Fill in the blanks

The data in the table actually represents the number of points Kobe Bryant scored in each game of the 2010 NBA Finals. The input is the game number and the output is the number of points he scored in each game.
6. Explain why there are seven numbers in the domain.
7. What is the maximum value of the function? What does this value represent? What does this imply about the scenario?
8. If the situation had gone differently, what is the minimum number of values that could have been in the range. How could this have been possible?
9. Explain why the real scenario guarantees that this relation would be a function.
10. Is this also true for the scenario you created in \#5? Explain.


## 1.6: Understanding Graphs

1. The graph below shows Mr. Goza's distance from home throughout the day, starting from 10:00 AM when he left his house.

a) Describe the domain and range of the graph using numbers and units.
b) At what time did Mr. Goza reach his furthest destination?
c) Does this graph represent a function? How do you know?
d) Is this graph discrete or continuous? Explain.
e) Write an equation to match the function.

Name:
2) Write a scenario or story to match the graph below. Make sure to appropriately label the axis using numbers and units.


Story:

Write an equation to match the function and your story.

Using numbers and units describe the domain and range of this function as accurately as possible.

## Quiz \#1: Functions

1. For the first week (5 days) of school, how many minutes per day did you spend on your homework? Fill in the chart with the information.

|  | 1 | 2 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |

2. Represent this relation with a...
a) table
b) mapping

## c) set of ordered pairs


3. What is the domain of the relation?
4. What is the range of the relation?
5. What is the minimum value of the relation? What does this value represent?
6. Is this relation a function? Why or why not?
7. Is the relation continuous or discrete? Explain how you know?
8. Draw a mapping of a relation that is not a function.
9. Draw a possible graph of a function that is continuous with a domain $[0,10]$.

## 1.6b More Functions Practice

1) On the plane below draw a relation that is not a function. Explain how you know.

2) Show the relation $(-2,-8),(-1,-1),(0,0),(1,1),(2,8)$ with a $\ldots$
a) mapping
b) graph

c) What is the domain of the relation?
d) What is the range of the relation?
3) On the plane below, draw a graph to match the given scenario. Make up units and numbers for the horizontal and vertical axis.

The amount of violent crimes in our neighborhood was increasing for months.
Then during the summer Mr. Goza moved in and the number of crimes decreased dramatically. We suspect that he is a Superhero.

a) What is the domain?
b) What is the range?
c) What is the maximum value of the function? What does it represent?
d) Write an equation to match the function.

## 1.7: Creating Graphs to Model a Scenario

Use the following information to create graphs on the graph paper attached. Make sure that your horizontal axis is always represents some unit of time. Any information that is not given can be made up by you! Be creative!

1) The height above ground level of a man who enters the elevator on the six floor, rides to the underground parking lot to get his laptop from his car and returns to the fourth floor for a meeting.
2) The number of people at the Staples Center starting two hours before a Lakers game and ending two hours after the game.
3) The temperature outside on a winter day in the mountains.
4) The height above sea-level of a piece of bait on a deep sea fisherman's line as he fishes in the Pacific Ocean (from bait to catch).
5) The approximate number miles you walked/ran on each of the last ten days.
6) The speed of a soccer ball that has been kicked straight up in the air, starting the moment it is kicked and ending the moment it hits the ground.

For each problem sketch a graph that matches the scenario described. Next to each graph describe why your graph looks the way it does. Try to use the words increasing, decreasing, and constant. Also, estimate the domain and range of your graph. Make sure to label your axis with both units and numbers. Lastly, provide equations each of the functions you graph (example: height $=f$ (time)).


1) Explanation:

Domain:
Range:

3) Graph:


Domain:
3) Explanation:

Domain:

Range:

Range:
4) Graph:



4) Explanation:

Domain: Range:
5) Explanation:

Domain:
Range:
6) Explanation:

Domain:
Range:

## 1.7b: More Graph Stories

Graph each scenario on a piece of graph paper. Include domain and range, name your functions, and provide equations for the graphs (ex: height $=f($ time $)$ ).

1) The heart rate, in beats per minute, of a runner from 5 minutes before she starts her run until a half hour after her run is complete.
2) The approximate number of text messages you sent in each month of this year.
3) The height (in feet of elevation) of a snowboarder from the time he gets in line for the ski lift to the time he gets back to the bottom of the mountain.
4) The distance from home of a person who has gone out for the day. (You make up the details of the story.)
5) The height above Earth of a UFO that comes down from outer-space, hovers above Earth just long enough to abduct Kanye West, and takes off back to another galaxy.

## 1.8: Dwayne Wade Signals Hello from a Distance

The following table is shows data of an unknown relation:

| $h$ | T |
| :---: | :---: |
| 0 | 95 |
| 1 | 97 |
| 3 | 100 |
| 6 | 105 |
| 7 | 104 |
| 10 | 100 |
| 12 | 97 |

1. Represent the relation with a mapping. 2. Represent the relation with a graph.

2. Is this a function?
3. What is the domain?
4. What is the range?

We can now say that $T$ is a function of $h$. Then we can say that $T=f(h)$. Or we can say the input is $h$ and the output is $f(h)$.
6. Is $f(h)$ discrete or continuous? How do you know?
7. What is the maximum value of $f(h)$ ? How do you know?
8. Find the average rate of change of $f(h)$ on the interval $[3,7]$.
9. Find the average rate of change of $f(h)$ on the interval $[6,12]$.

The reason that this assignment was called "Dwayne Wade Signals Hello from a Distance" is because the data in the table represents the temperature in Miami (starting at 8:00 AM and ending at 8:00 PM) during a Heat Wave. (Get it? Miami Heat Wave?). Here is a picture of the actual weather from that day.


Think about and discuss with your group: How is the data from the table on the first page consistent with the temperatures shown in the weather report above?

Knowing that $f(h)$ represents the temperature in Miami throughout a given day changes our answers from \#2 - \#6 significantly and gives us insight to what our answers mean. With a colored pen or pencil go back and re-answer 2-6. Don't erase your old answers!
10. Write an equation to match the function now that you know what it is about.
11. Does the new information about $f(h)$ change our answer to \#7? Why or why not?
12. Now that we know $f(h)$ is about the temperature in Miami, explain what your answer to \#8 means in terms of the situation.
13. Explain why your answer to $\# 9$ is negative based on the situation.
14. What is the average rate of change of $f(h)$ on the interval $[1,12]$. Why is this answer deceiving/confusing?
15. Explain why any relation where the input is time and the output is an outdoor temperature is going to be a function.
16. Explain why any function where the input is time and the output is an outdoor temperature is going to be continuous.

## 1.9: Vocabulary Practice

Easy: On a separate sheet of paper create a possible example of each:

1. A mapping of a relation that is not a function.
2. A set of ordered pairs of a relation that is a function.
3. A discrete graph of a relation that is not a function.
4. A graph of a relation that is a continuous function.
5. A table of a relation that is a function.

Harder: For each problem 6-12 create a graph for a function ...
6. with a maximum value 20 and a minimum value of 5 .
7. with a domain $[0,6]$ and a range $[-5,4]$.
8. where the average rate of change on $[0,5]$ is 2 .
9. that is discrete with a story to match.
10. that is always decreasing with a story to match.
11. made up of 4 lines with different Rates of Change.
12. that is constant on the interval $[5,9]$ with a story to match.

### 1.10 Functions in the LBC

I recently took a trip down to Long Beach to see Jerry Seinfeld perform stand-up comedy. From the time I left my house at 5:00 PM, I kept data on how far I was from my home (shown in the table below). On the way there I picked my wife up from work, we got some drive-thru for dinner, fought some traffic and then got lost trying to park. The show was very funny so it made the trip worthwhile.

| Hours <br> sinee <br> $5: 00$ PM | Milesfrom <br> home |
| :---: | :---: |
| 0 | 0 |
| 0.5 | 10 |
| 1 | 15 |
| 2 | 30 |
| 2.5 | 35 |
| 4 | 35 |
| 4.5 | 35 |
| 5 | 0 |

Let's think about the function $f(h)$ that will output my distance from home when a given number of hours since 5:00 PM is put in.

1) Write an equation to match this function.
2) True or False: $f(h)$ is a position function. Explain your answer.
3) True or False: $f(h)$ is a continuous function. Explain your answer.
4) How far was I from home at 5:30 PM. How do you know?
5) What is $f(2)$. What does your answer represent?
6) Estimate $f(3)$. Justify your estimation.
7) How long do you think the comedy show was? What evidence do you have for this?
8) About how many miles is it from my house to Long Beach? How do you know?
9) Sketch a possible graph of $f(h)$ given what we know about the function.

10) What is the domain of $f(h)$ ?
11) What is the range of $f(h)$ ?
12) On what interval is $f(h)$ increasing? How does that match my story?
13) On what interval(s) is $f(h)$ constant? How does that match my story?
14) On what interval is $f(h)$ decreasing? How does that match my story?

The rate of change of position is called "velocity."
15) What was my average velocity on the way to Long Beach?
16) What was my average velocity on the way home?
17) Explain why the answer from \#16 is negative.
18) Was I driving faster on average on the way to Long Beach or on my way home? How do you know?
19) What was my average velocity on the interval $[2,2.5]$. How does this match my story?
20) Estimate my velocity at exactly 5:30. How did you come to this conclusion?

### 1.10b Review: More Functions Practice



1) Is the relation shown in the graph a function? Why or why not?
2) Represent the relation in the graph with a...
a) Table
b) Mapping
3) Is the relation continuous or discrete? How do you know?
4) What is the domain of the relation? 5) What is the range of the relation?

Assume that the function graphed below represents a runner's speed during a 100 meter sprint from the time the race starts to the time she comes to a rest at the end.


1) Is this function continuous or discrete? How do you know?
2) What is $f(2)$ ? What does your answer tell us?
3) What is the domain of this function?
4) How many seconds do you think it took for the runner to complete the race? How did you come to this conclusion? (Your answer should not be 18 seconds!)
5) What is the maximum value of the function? What does it represent?
6) What does an output of zero represent in this scenario?

### 1.11: Advanced Graph Stories

For each of the following scenarios create the graph of a function with the given input and output to match the situation that is described. Include domain, range, and an equation for each of the functions you graph.

1) I walked to a friend's house that was 6 miles away. On the way my average velocity was 2 mph . I stayed for 4 hours. On my way home I was in a bit of a hurry so I walked faster (with an average velocity of -4 miles per hour).
Input: time
Output: distance from my house


Domain:

Range:

Explanation:

Equation:
2) Starting from the top of a 12,000 foot high mountain a snowboarder rides down 3,000 feet. He stops there to catch a lift to the top of another mountain that is even higher than the one he just rode. He waits 5 minutes in line at the lift before getting on. This lift is slow and it takes a while to get to the top of the mountain. Input: time (since he started down the first mountain)

Output: Elevation


Domain:
Range:
Equation:
3) Over the last ten days I've been trying to run more. I've only taken 3 days off. The most miles I ran in one day was 7 , the least (besides days off) was 2. I would typically take days off after longer runs.
Input: days
Output: miles ran


Domain:
Range:
Explanation:
4) A scuba diver jumps from his boat into the ocean. He proceeds to dive 100 feet below sea level in 2 minutes. He stays there looking at an eel for 2 minutes then dives deeper. After 15 minutes he heads back up to the boat at a quickened pace because he fears that there are sharks nearby.
Input: time Output: height above sea level


Domain:
Range:

Equation:

## Unit 1 CYO: Create Your Own Function Graph

In class we did a set of problems where you were asked to draw a graph to match a situation that was described on the worksheet. Now I am going to ask you to make up your own situation and draw a graph to go with it.

Requirements (you will receive a zero if these are not met)

- Assignment must be done on poster size paper.
- Your poster must include a graph with a description of the situation/phenomena/story being represented.
- Your poster must be turned in by $\qquad$ .
- You may work individually or with one partner of your choice.


## Rubric

I will be checking to see if:

- The curve you draw accurately matches the situation/phenomena/story you describe. The graph is clear and the description is well written. >> $\mathbf{1 0}$ points
- Your units and labels for your axis are clear, correctly scaled, numbered, and make sense according to your description. >> 3 points
- The situation you describe applies to a real life or fictitious situation that actually makes sense. >> 2 points
- The subject matter of your poster is creative and/or interesting. >> 3 points
- The poster looks presentable and all work/thought is clearly shown. >> 3 points
- The domain and range is included and correctly given. >> 2 points
- Your function is named and the equation for the graph is given. >> 2 points

I look forward to laminating some of these for permanent display. Extra credit will be given to posters that get laminated. There is a lot of freedom here. Be creative and make them look good!

This project will be worth $\mathbf{2 5}$ points and will go into your tasks category.

### 1.12: Riding to the Lake

## PART 1: Consider thefollowing..

This summer I spent 3 weeks in Park City, Utah which has a great system of bike paths. I decided to ride one of the paths to a lake. The lake was ten miles away on the path from the hotel where I was staying. I left at 12:00 PM and planned to return by 4:00 PM to get ready to go out for dinner with some friends.

Assume $f(t)$ is a position function that measures my distance from the hotel from the time I left to the time I returned where $t$ is measured in hours since 12:00 PM.

Sketch one possible graph of $\boldsymbol{f}(\boldsymbol{t})$ to represent my trip to the lake on the axis shown below. Provide a detailed story to match your graph. Then answer the ten questions on the following page. Since you will be using your graph to answer the questions it is recommended that you read the ten questions before creating your graph and story. (7 points possible)


Story:

Use your graph of $f(t)$ to answer the following questions:

1) What is the domain $f(t)$ ? (2)
2) What is the range? (2)
3) According to your graph, at what time did I get to the lake? (2)
4) According to your graph, what is $f(1)$ ? What does this answer represent? (2)
5) According to your graph what was my average velocity on the way to the lake? (3)
6) According to your graph, on which interval(s) was $f(t)$ decreasing? How does that match your story? (2)
7) Is your version of $f(t)$ continuous or discrete? Explain why. (2)
8) What was the maximum value of $f(t)$. What does that represent in the story? (2)
9) We know $f(t)$ is a function. Explain why this is obvious based on what it represents in the real world. (2)
10) For each part of this problem you will get one point for an answer that makes sense, one point for a justification of your answer, and one point for correctly making use of one of the vocabulary words listed below in your justification:

| domain | constant | maximum |
| :--- | :--- | :--- |
| range | velocity | minimum |

## How would (or why wouldn't) your graph of $f(t)$ change if I told you...

a) I went past the lake to see a waterfall further down the path? (3)
b) I stayed at the lake too long and got back late after hurrying back? (3)
c) I went swimming in the lake? (3)

## PART 2: Complete the graph and the table using the given information.

In Part 1 of this Task you were asked to make up information about my ride to the lake. However, the real details of that ride had not been revealed. Since it was a while back I can't remember all the details either, but here's what I was able to remember:

- My average velocity during the first 30 minutes was 10 miles per hour.
- 45 minutes after I left I stopped for 5 minutes to take pictures of some wildlife.
- At 1:00 I was 8 miles along the bike path ( 2 miles from the lake).
- I arrived at the lake at 1:30.
- I spent one hour at the lake.
- I never went further than 10 miles from my hotel.
- From 2:30 to 3:30 I was riding back with an average velocity of 8 miles per hour.
- At 3:00 exactly I was riding faster than I had all day (around 15 mph ).
- 3.5 hours after I left I stopped to rest for 15

| Minutes <br> (after 12:00) | Miles <br> (from hotel) |
| :---: | :---: |
| 0 |  |
| 30 |  |
| 60 |  |
| 90 |  |
| 150 |  |
| 210 |  |
| 225 |  |
| 240 |  | minutes.

- I got back to the hotel right on time.



## PART 3: People go nuts for Goza's Donuts.

At Mr. Goza's Donut Shop single donuts are $\$ 1$ and a dozen (12) donuts cost $\$ 8$.
11) How much would it cost for 5 Goza's donuts? (1)
12) How much would it cost for 25 Goza's donuts? (2)
13) Your teacher doesn't know about the special price for a dozen donuts and gives you $\$ 40$ to get 40 donuts for your class. She tells you if there is any change you can keep it for being so kind and picking up donuts for the class. How much money should you come away with? (2)
14) Assume you are going to buy 22 donuts.
a) What is the cheapest way to buy 22 donuts? How much will it cost? (2)
b) List all the different ways you could buy 22 donuts and the cost of each way.
15) Complete the graph for purchasing 0 to 24 donuts. Consider all possible ways to buy them in your graph. (3)

16) Is the relation in the graph a function? Explain why or why not. (2)

