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Session 473
PSCC –Mesquite F
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Arithmetic Sequences and Series as a Bridge to Quadratics

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Sample Lessons

1) Task 3.1: Push Up Duck
2) Task 3.5: Hot Cheetos Craze
3) Task 3.6: Finding a Shortcut
4) Unit 3 Exam Part 2: The Card Tower

*The complete Unit 3 is online at *www.coast2coast.me/nate*

3.1: Push Up Duck

Football games at the University of Oregon are always exciting. The Oregon Ducks are known for scoring points, and their mascot is known for keeping track of those points by doing pushups. At the conclusion of each of Oregon's scoring drives the Duck does a pushup for every point the team has scored in the game up to that point. In a high scoring game, this can add up fast!

Watch this clip for info on the Duck: http://www.youtube.com/watch?v=5ve92hOixGo

In 2012, Oregon scored 70 points while beating conference rival Colorado! Here is information from the game: http://scores.espn.go.com/ncf/boxscore?gameId=323012483

GAMEHQ
Scores Pac-12

Final	Final	Final	Final	Final	Final
UCLA 45	COLO 14	10 USC 36	WSU 17	CAL 27	7 ORST 17
ASU 43	2 ORE 70	ARIZ 39	19 STAN 24	UTAH 49	WASH 20

Scoring Summary

FIRST QUARTER				COLO	ORE
O	TD	13:13	Kenjon Barner 1 Yd Run (Rob Beard Kick)	0	7
O	TD	12:28	De'Anthony Thomas 9 Yd Run (Rob Beard Kick)	0	14
O	TD	06:56	Marcus Mariota 5 Yd Run (Rob Beard Kick)	0	21
O	TD	01:44	Bralon Addison 16 Yd Pass From Marcus Mariota (Rob Beard Kick)	0	28
SECOND QUARTER				**COLO**	**ORE**
O	TD	12:36	Kenjon Barner 24 Yd Run (Rob Beard Kick)	0	35
O	TD	11:06	De'Anthony Thomas 73 Yd Punt Return (Rob Beard Kick)	0	42
O	TD	06:16	Daryle Hawkins 7 Yd Pass From Marcus Mariota (Rob Beard Kick)	0	49
O	TD	00:20	Bryan Bennett 6 Yd Run (Rob Beard Kick)	0	56
THIRD QUARTER				**COLO**	**ORE**
CU	TD	09:41	Christian Powell 1 Yd Run (Will Oliver Kick)	7	56
CU	TD	08:02	Christian Powell 20 Yd Run (Will Oliver Kick)	14	56
O	TD	03:55	Bryan Bennett 3 Yd Run (Rob Beard Kick)	14	63
O	TD	01:00	Bryan Bennett 17 Yd Run (Rob Beard Kick)	14	70

Team Stat Comparison

	COLO	ORE
1st Downs	12	30
3rd down efficiency	2-13	8-12
4th down efficiency	2-3	1-1
Total Yards	246	617
Passing	96	192
Comp-Att	13-27	14-18
Yards per pass	3.6	10.7
Rushing	150	425
Rushing Attempts	33	57
Yards per rush	4.5	7.5
Penalties	5-60	7-85
Turnovers	3	1
Fumbles lost	2	1
Interceptions thrown	1	0
Possession	32:13	27:47

Answer the following questions regarding Oregon's 70-14 victory against Colorado.

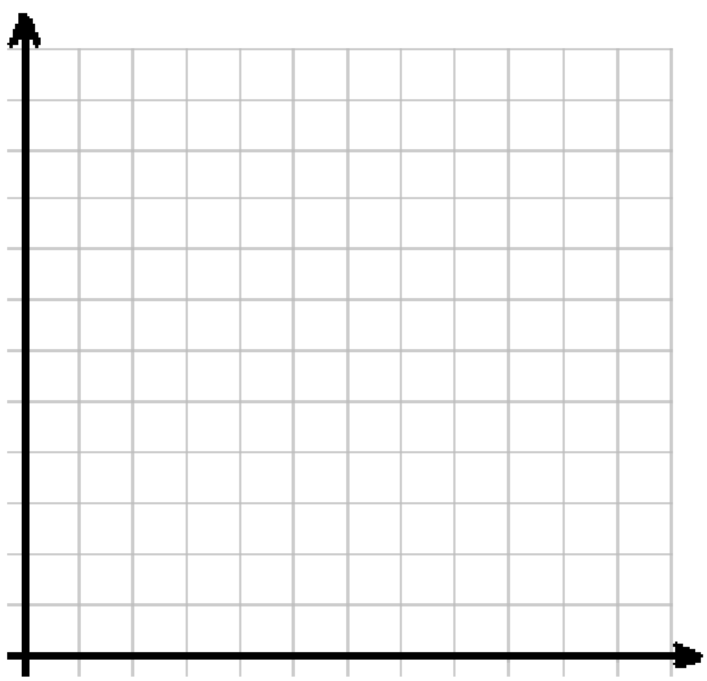
1) How many pushups *did the Duck do* after the first touchdown?

2) How many pushups *did the Duck do* after Oregon scored its third touchdown?

3) How many touchdowns did Oregon score in the Colorado game?

Assume that the function *D(n)* models the number of pushups the Duck did after Oregon scored its n^{th} touchdown during their game against Colorado.

4) Graph the equation ***# of Pushups = D(n)*** on the axis below.



5) True or False: *D is a linear function*. Explain your choice.

6) Is this function continuous or discrete? Explain why?

7) What is the domain of *D*?

8) What is the range of *D?*

9) What is *D(8)*? What does it represent?

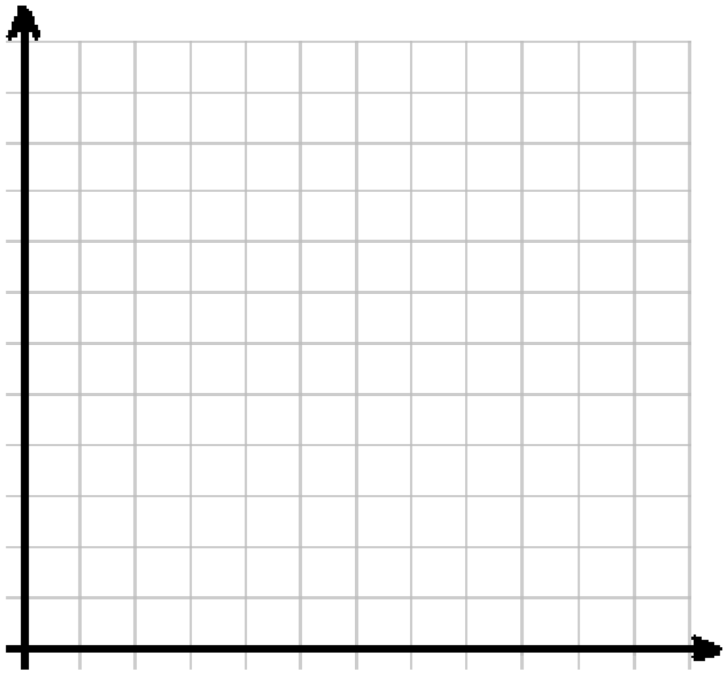
10) How many ***total pushups had the Duck done*** after Oregon scored 3 touchdowns?

11) At Halftime, Oregon had scored 56 points. How many total pushups had the Duck done by then?

12) What was the total number of pushups done by the Duck in the game against Colorado?

Let's say the function *T(t) models the* ***total number of push-ups*** *the Duck had done after Oregon had scored t touchdowns.*

13) Use the axis below to graph the equation ***Total # of Pushups = T(t)***. You may want to make a table to keep track of the ordered pairs.



14) True or False: *T is a linear function*. Explain your choice.

15) Using **Sigma Notation**, write an expression to represent the total number of push-ups the Duck had done after Oregon had scored *t* touchdowns.

3.5: Hot Cheetos Craze

Kids these days love Hot Cheetos. Last time I saw an Ortho student at Target, this was their cart:

Ms. Weeks, who fears the worst, thinks kids are eating more and more junk food. She predicts that a typical Ortho student eats their first 10 small bags of Hot Cheetos in 1st grade, eats 30 small bags in 2nd grade, eats 50 small bags in 3rd grade and continues this pattern of additional consumption until they graduate high school.

1) How many bags of Hot Cheetos does a given student eat in 5th grade, according to Ms. Weeks?

2) The number of Hot Cheetos a student eats each year can be thought of as a sequence. Write the first 5 terms of the sequence.

3) What type of sequence is this? How do you know?

4) Find the explicit formula ($a_n =$ _____) that matches this sequence.

5) According to your formula what is a_9? What does it represent in this scenario?

6) How many small bags of Hot Cheetos does Ms. Weeks predict a typical Ortho student will eat in his/her senior year? Does this sound reasonable? Why or why not?

7) According to Ms. Weeks, how many total bags of Hot Cheetos will a student have eaten by the time they graduate from Ortho?

Mr. Warren says that, without proper health education, this pattern of Hot Cheeto consumption can continue far beyond high school graduation. Say a student continued to increase their Cheetos eating for sixty years (until about the time he/she retired).

8) Does Mr. Warren's fear seem reasonable to you? Why or why not?

9) Create an expression using Sigma Notation to find how many small bags of Hot Cheetos the student will have eaten by the time they retire after a total of 60 years of Hot Cheetos eating.

10) Explain why no one wants to evaluate the expression in #9.

11) Take a few minutes to estimate the sum from #10. Show some mathematical evidence or reasoning that supports your estimate.

3.6: Finding the Shortcut

Lets think about the most basic twenty term Arithmetic Series out there: the on that adds the first 20 “natural numbers”.

$1 + 2 + 3 + \ldots$

1) What rule matches the sequence 1, 2, 3, …

2) Use sigma notation to create an expression that represents the sum of the first 20 terms of this sequence.

3) Instead of adding the numbers one by one, write out the entire sum so we can look for patterns that will make it easier to add. Be creative. Do you see anything?

4) Add the first and the last term of the sum. What do you get?

5) Add the second and second to last term. What do you get?

6) How many such pairs are there?

7) What is the sum of the first 20 terms? (Don't add them one by one!)

8) This will help you later:

Explain where the 21 came from.

Explain where the 10 came from.

9) Quick, let's try another: $\sum_{n=1}^{20} 2n+4$ using the same pattern.

What is the sum of the first and last term?

How many such pairs are there?

What is the sum of all twenty terms?

10) Explain how to find the sum of the first 20 terms of any Arithmetic Series.

11) Let's use a similar process to evaluate $\sum_{n=1}^{50} 2n$.

What is the sum of the first and last term?

How many such pairs are there?

What is the sum of all 50 terms?

12) How is this similar to the 20 term series?

13) Generalize what you did in #9 and #11 to explain how to find the sum of the first *k* terms of any Arithmetic Series.

14) Can you write a formula for $\sum_{n=1}^{k} a_n$? It's possible and would be cool if you do!

15) Use your formula to evaluate:

a) $\sum_{n=1}^{30} 3n+5$

b) $\sum_{n=1}^{100} -2n+10$

3.7: The Odd Addition

1) Think about the sequence of odd natural numbers starting with one. Write the first 5 terms of that sequence. What kind of sequence is it?

2) Use the terms in number 1 to complete the table below:

x	The sum of the first x odd natural numbers
1	
2	
3	
4	
5	

3) What pattern do you notice in the table?

4) Use the pattern to predict the sum of the first 10 odd numbers.

5) Say that there is a function T such the $T(x)$ is the sum of the first x odd natural numbers. Then according to the table…

$T(x)$ = _______

The result in #5 is simple and fantastic. The sum of the terms in the sequence 1, 3, 5, … always adds to the square of the number of terms. $T(x) = x^2$ is the most basic ***Quadratic Function***, a type of function we will explore further. Let's take a minute to prove this equation to be true.

6) Write a rule to match the sequence 1, 3, 5, …

$a_n =$

7) Use summation notation to create an expression that represents the sum of the first 20 terms of the series 1 + 3 + 5 + …

8) Use the equation for the sum of the first *k* terms: $\sum_{n=1}^{k} a_n = \frac{k}{2}(a_1 + a_k)$ to evaluate the expression you created in #7.

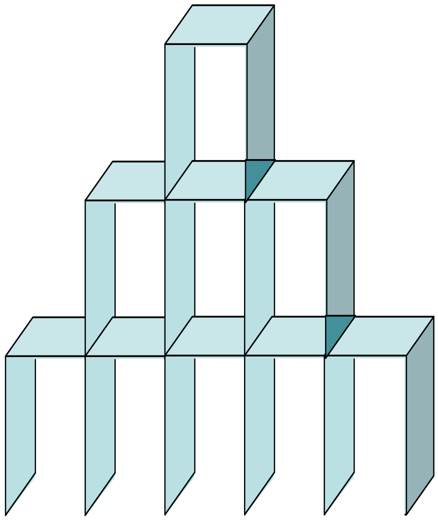
9) Use the same process from #7 & #8 to write a simplified expression for the sum of the **first *x* terms** of the series 1 + 3 + 5 + … (Hint: Set *k* = *x* and simplify the resulting expression.)

Your answer to #9 should be the same answer you got in #5. Check that it is. Hooray Math!

10) Using the function for #5, what is *T(100)*? What does the answer tell us?

Unit 3 Exam Part 3: The Card Towers

Mr. Tran and Mr. Goza have each begun constructing card houses as shown below. Mr. Tran, who loves triangles, has created the structure on the left, and Mr. Goza has opted for rectangular shapes as shown on the right.



1) Who uses more cards to create a 4 story version of their tower? Justify your answer with mathematical evidence. <u>(3)</u>

2) How many cards would it take Mr. Tran to make his card structure

a) 10 stories tall? <u>(1)</u>

b) 100 stores tall? <u>(2)</u>

3) If Mr. Goza builds his structure to a height of 20 stories, how many cards will he need *for he bottom level*? (2)

4) Create a quadratic function *C* where *C(x)* matches the number of cards in Mr. Goza's structure when it is *x* stories tall. (2)

5) Use factoring to find out how many stories Mr. Goza's structure will have if he uses 210 cards to make it. (Hint: (20)(21) = 420) (2)

6) How many more ***stories taller*** will Mr. Tran's structure be than Mr. Goza's if each of them are given 3 decks of cards to make their structure. Assume each deck has 52 cards. (4)